Problem 11.1

a) \( \langle (a + b)^2 \rangle = \langle a^2 \rangle + \langle b^2 \rangle + 2\langle ab \rangle \) where the DC terms \( \langle a^2 \rangle + \langle b^2 \rangle \) can be subtracted, leaving the time average of the product of the two signals \( a(t) \) and \( b(t) \).
Q.E.D.

b) \( \langle (a + b)^2 \rangle + \langle (a + b)^3 \rangle = \langle a^2 \rangle + \langle b^2 \rangle + 2\langle ab \rangle + c[\langle a^3 \rangle + 2\langle a^2 b \rangle + 2\langle ab^2 \rangle + \langle b^3 \rangle] \). All the cubed terms average to zero, so a non-ideal diode with odd terms in the diode characteristic still functions as an ideal multiplier. The higher-order even powers are of greater potential interest, but the ability of the square term plus all higher order odd terms to approximate the local i-v characteristic of a diode leaves little room for producing much DC energy due to other terms. Only in highly sensitive systems is this a typical concern.

Problem 11.2

Referring to Equation (5.2.5) and Figure 5.2-2 in the text, we see that \( \langle ab \rangle \) is the fringe modulation envelope, which is proportional to the field correlation function \( \phi_E(d/c) \), where \( d \) is the differential distance traveled by the two rays, analogous to \( L \sin \phi \) in the figure and \( c \) is the velocity of light. Note that the field \( E(t) \) to which \( \phi_E(\tau) \) refers is the slowly varying envelope of the sine wave of interest, and so its bandwidth corresponds to the bandwidth \( B \) of the optical signal:

\[
3 \times 10^8 / 10^7 - 3 \times 10^7 / 6 \times 10^{-7} = 10^{14} \text{ Hz}
\]
and its power density spectrum \( S_E(f) \) has the form:

\[
\phi_E(\tau) \approx \frac{B^2}{2}
\]

Since \( \phi_E(\tau) \), we have:
and the half-power point on \( \phi_E(\tau) \) is approximately \( \tau = 1/3B = d/c \). Therefore
\( d \approx c/3B = 3 \times 10^{14} / 3 \times 10^{14} = \) one micron.

Problem 11.3

This Hanbury-Brown-Twiss interferometer deserves \( \phi_E(\tau) \)

Referring to (5.2.15) we see that \( \phi_E(\tau) \) \( \leftrightarrow \) \( |E(\psi)|^2 \propto I(\psi) \), as illustrated. It follows that the first null in the 2-D sinc function \( \phi_E(\tau) \) falls at \( 10^7 \) wavelengths, or \( 10^7 \times 0.5 \times 10^{-9} = 5 \text{ mm.} \)