Lecture 2 - Carrier Statistics in Equilibrium

February 8, 2007

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Reading assignment:

del Alamo, Ch. 2, §§2.1-2.4 (2.4.1)

Announcement:

Go to http://ilab.mit.edu and register. Select membership in the 6.720 group. You will need this to access the lab for the Device Characterization Projects.
Key questions

• What are these "energy band diagrams"?
• What are these "holes"?
• In a perfectly pure semiconductor, how many electrons and holes are there?
• Can one engineer the electron and hole concentrations in a semiconductor?
1. Conduction and valence bands, bandgap, holes

Conduction and valence bands:

- *bonding electrons* occupy states in valence band
- "free" *electrons* occupy states in conduction band
- *holes*: empty states in valence band
- CB electrons and VB holes can move around: "*carriers*"
Elements of energy band diagrams:

- at edges of bands, kinetic energy of carriers is zero
- electron energies increase upwards
- hole energies increase downwards
2. Intrinsic semiconductor

Define *intrinsic semiconductor*, or ”ideal” semiconductor:

• perfectly crystalline (no perturbations to periodic lattice)
• perfectly pure (no foreign atoms)
• no surface effects

Question: *How many electrons and holes are there in an intrinsic semiconductor in thermal equilibrium at a given temperature?*

Answer requires fairly elaborate model [lecture 3], but key dependencies can be readily identified.

Define:

\[ n_o \equiv \text{equilibrium (free) electron concentration in conduction band} \ [cm^{-3}] \]

\[ p_o \equiv \text{equilibrium hole concentration in valence band} \ [cm^{-3}] \]

Certainly in intrinsic semiconductor:

\[ n_o = p_o = n_i \]

\[ n_i \equiv \text{intrinsic carrier concentration} \ [cm^{-3}] \]
Key dependencies of $n_i$:

- **Temperature**:

  $$T \uparrow \Rightarrow n_i$$

- **Bandgap**:

  $$E_g \uparrow \Rightarrow n_i$$

What is detailed form of dependencies?

Use analogy of chemical reactions.
Electron-hole formation can be thought of as chemical reaction:

\[ \text{bond} \rightleftharpoons e^- + h^+ \]

similar to water decomposition reaction:

\[ H_2O \rightleftharpoons H^+ + OH^- \]

*Law-of-mass action* relates concentration of reactants and reaction products. For water:

\[
K = \frac{[H^+][OH^-]}{[H_2O]} \sim \exp\left(-\frac{E}{kT}\right)
\]

\(E\) is energy consumed or released in reaction.

This is a "thermally activated" process \(\Rightarrow\) rate of reaction limited by need to overcome energy barrier \(E\) (*activation energy*).

In analogy, for electron-hole formation:

\[
K = \frac{n_0p_o}{[\text{bonds}]} \sim \exp\left(-\frac{E_g}{kT}\right)
\]

[\text{bonds}] is concentration of unbroken bonds.

Note: activation energy is \(E_g\).
In general, relatively few bonds are broken. Hence:

\[ \text{[bonds]} \gg n_o, p_o \]

and

\[ \text{[bonds]} \sim \text{constant} \]

Then:

\[ n_op_o \sim \exp(-\frac{E_g}{kT}) \]

Two important results:

- First,

\[ n_i \sim \exp\left(-\frac{E_g}{2kT}\right) \]

As expected: \( T \uparrow \Rightarrow n_i \uparrow \)

\( E_g \uparrow \Rightarrow n_i \downarrow \)

To get prefactor, need detailed model [lecture 3].
Arrhenius plot for Si [experiments of Misiakos and Tsamakis, 1993]:

In Si at 300 K, \( n_i \approx 1.1 \times 10^{10} \, \text{cm}^{-3} \).

- Second important result:

\[
n_0 p_0 = n_i^2
\]

*Equilibrium np product in a semiconductor at a certain temperature is a constant specific to the semiconductor.*
3. Extrinsic semiconductor

Can electron and hole concentrations be engineered?

Insert *dopants* in substitutional positions in the lattice:

- **Donors**: introduce electrons to conduction band without introducing holes to valence band
- **Acceptors**: introduce holes to valence band without introducing electrons to conduction band

If any one carrier type overwhelms $n_i \Rightarrow$ extrinsic semiconductor
Donor in Si, atom from column V (As, P):

- a) neutral donor
- b) ionized donor

Acceptor in Si, atom from column III (B):

- a) neutral acceptor
- b) ionized acceptor
Representation of donor and acceptor states in energy band diagram:

\[ E_d, E_a \sim 40 - 60 \text{ meV}, \text{ for common dopants} \]

\[ N^+_D \simeq N_D \]

\[ N^-_A \simeq N_A \]

Typical doping levels:

\[ N_A, N_D \sim 10^{15} - 10^{20} \text{ cm}^{-3} \]
\( \square \) n-type semiconductor

\[
\begin{align*}
n_o & \simeq N_D \\
p_o & \simeq \frac{n_i^2}{N_D}
\end{align*}
\]

These equations are valid at intermediate temperatures.

\( \square \) p-type semiconductor

\[
\begin{align*}
p_o & \simeq N_A \\
n_o & \simeq \frac{n_i^2}{N_A}
\end{align*}
\]
4. Conduction and valence band density of states

Close to edges:

\[ g_c(E) \propto \sqrt{E - E_c} \quad E \geq E_c \]

\[ g_v(E) \propto \sqrt{E_v - E} \quad E \leq E_v \]

MIT OpenCourseWare (http://ocw.mit.edu/), Massachusetts Institute of Technology. Downloaded on [DD Month YYYY].
Common expressions for DOS:

\[
g_c(E) = 4\pi \left(\frac{2m_{de}^*}{h^2}\right)^{3/2} \sqrt{E - E_c} \quad E \geq E_c
\]

\[
g_v(E) = 4\pi \left(\frac{2m_{dh}^*}{h^2}\right)^{3/2} \sqrt{E_v - E} \quad E \leq E_v
\]

\[m_{de}^* \equiv \text{density of states electron effective mass}\]
\[m_{dh}^* \equiv \text{density of states hole effective mass}\]
Key conclusions

• Concept of *(free)* electron: electron in conduction band.
• Concept of hole: empty state in valence band.
• Intrinsic semiconductor: ideally pure semiconductor.

\[ n_o = p_o = n_i \sim \exp\left(-\frac{E_g}{2kT}\right) \]

• To first order, for a given semiconductor \( n_o p_o \) is a constant that only depends on \( T \):

\[ n_o p_o = n_i^2 \]

• Equilibrium carrier concentrations can be engineered through shallow dopants ⇒ extrinsic semiconductor.
  
  – n-type semiconductor:

\[ n_o \sim N_D, \quad p_o \sim \frac{n_i^2}{N_D} \]

– p-type semiconductor:

\[ p_o \sim N_A, \quad n_o \sim \frac{n_i^2}{N_A} \]

• Around edges, conduction and valence bands in semiconductors feature \( DOS \sim \sqrt{E} \).

• Order of magnitude of key parameters for Si at 300 K:

  – intrinsic carrier concentration: \( n_i \sim 10^{10} \text{ cm}^{-3} \)

  – typical doping level range: \( N_D, N_A \sim 10^{15} - 10^{20} \text{ cm}^{-3} \)
Self study

- Charge neutrality