Lecture 17: Flux Flow & Pinning

Outline

1. Review of Vortices
2. Flux Flow
3. Pinning
4. Critical State Model

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Fluxoid Quantization and Type II Superconductors

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The Vortex State

\[ \langle B \rangle = n_V \Phi_V \]

\( n_V \) is the areal density of vortices, the number per unit area.

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Please see: "A current-carrying type II superconductor in the mixed state" from \url{http://phys.kent.edu/pages/cep.htm}
Vortices in High-Field Magnets

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Why use Type-II for Magnets?

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Vortex Driven by a Uniform Current

Currents add
\[ J > J_C \]

Currents subtract
\[ J > J_C \]

\[ J \times B \]

Power Dissipation due to Flux Flow

Let the vortices move in unison with velocity \( u_c \)

\[ \Delta \Phi = \Phi_0 n_V h u_x \Delta t \]

Faraday’s law gives a voltage in direction of current

\[ v = \frac{d\Phi}{dt} = \Phi_0 n_V h u_x \]

The power dissipated is then

\[ P_{\text{dis}} = iv = i\Phi_0 n_V h u_x \]


We now seek a more microscopic description of the dissipation. This will allow us to find the velocity. So we look at one vortex to see where the power is dissipated.
Microscopic Picture of Dissipation

Near the core,
\[ J_V(r) = \frac{\Phi_0}{2\pi \lambda^2} \frac{1}{\sqrt{(x-u_x t)^2 + y^2}} \psi \]

The electric field in the superconductor is
\[ E^{\text{out}}(r) = \frac{\partial}{\partial t} \left( \mu_0 \lambda^2 J_V(r) \right) \]

So that from the chain rule
\[ E^{\text{out}}(r) = u_x \frac{\partial}{\partial x'} \left( \mu_0 \lambda^2 J_V(r) \right) \]
A little algebra gives a dipole field!
\[ E^{\text{out}}(r) = -u_x \frac{\Phi_0}{2\pi \beta^2} \left( i_{r'} \sin \phi' - i_{\phi'} \cos \phi' \right) \]

Therefore, the field inside the core must be a uniform field:
\[ E^{\text{in}}(r) = u_x \frac{\Phi_0}{2\pi \xi^2} i_y \quad \text{and} \quad J^{\text{in}}(r) = \sigma_o E^{\text{in}}(r) \]
Power dissipated in the Core

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The power dissipated is

\[ P_{\text{dis}}^{\text{core}} = \int_{\text{core}} E^{\text{ln}}(\mathbf{r}) \cdot J^{\text{ln}}(\mathbf{r}) \, dV \]

\[ P_{\text{dis}}^{\text{core}} \approx \frac{u_x^2 \Phi_0^2 \sigma_o}{4 \pi \xi^2} L_z = \frac{u_x^2 \Phi_0^2 \sigma_o}{G_L} \frac{2 \pi \xi^2}{L_z} \]

Adding the power from each vortex gives the total power dissipated

\[ P_{\text{dis}} = \frac{u_x^2 \Phi_0^2 \sigma_o}{4 \pi \xi^2} n_V 2ahL_z \]

Flux-Flow Resistance

\[ P_{\text{dis}} = i \Phi_0 n_V \hbar u_x \]

Macroscopic picture

\[ P_{\text{dis}} = \frac{u_x^2 \Phi_0^2 \sigma_o}{4 \pi \xi^2} n_V 2ahL_z \]

Microscopic picture

Comparing these results gives:

\[ u_x = \frac{2 \pi \xi^2}{\Phi_0 \sigma_o} \frac{1}{2aL_z} i \]

Therefore, we find that the power dissipated can be written as

\[ P_{\text{dis}} = i^2 R_{\text{ff}} \quad \text{where} \quad R_{\text{ff}} = \frac{\hbar}{2aL_z \sigma_o} 2\pi \xi^2 n_V \]

Flux-flow resistivity

\[ \rho_{\text{ff}} = \rho_o 2\pi \xi^2 n_V = \rho_o \frac{B}{\mu_0 H_c} \quad \text{as if current goes thru the cores} \]
Flux-flow viscosity

Total force can be thought of as a current driving force plus a viscous drag force

\[ f'_\text{tot} = f'_L + f'_d \]

\[ J \times \Phi_o i_z \quad -\eta u \]

\( f' \) is per unit length

The forces balance in steady state, so that

\[ u_x = \frac{J \Phi_o}{\eta} \]

We found before that

\[ u_x = \frac{2\pi \xi^2}{\Phi_o \sigma_n} \quad \text{i.e.} \quad \eta = \frac{\Phi_o^2}{2\pi \xi^2 \sigma_c} \]

How do we keep vortices from moving and dissipating energy?

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Pinning a Vortex

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a) Free energy decrease if core is in the normal region: \[ \Delta G_{\text{core}} = \frac{1}{2} \mu_0 H_c^2 \pi \xi^2 L_z \]
b) Free energy core is partly in the normal region: \[ \Delta G_{\text{core}} = \frac{1}{2} \mu_0 H_c^2 \pi \xi L_z \]

Therefore, there is a force restraining the vortex in the normal region given by

\[ f_p = -\frac{1}{2} \mu_0 H_c^2 \pi \xi L_z = \frac{\Phi_0^2}{16\pi \mu_0 \lambda^2 \xi} L_z \]

Critical Current Density

When the force of the applied current density equals the pinning force, vortices move:

\[ f_I = J_{\text{ext}} \times \Phi_0 L_z \]

\[ f_p = -\frac{\Phi_0^2}{16\pi \mu_0 \lambda^2 \xi} L_z \]

\[ J_{C_p}^{\text{max}} = \frac{\Phi_0}{16\pi \mu_0 \lambda^2 \xi} \text{ Depinning critical current processing dependent} \]

\[ J_{\text{pair}} = \frac{\Phi_0}{3\sqrt{3}\pi \mu_0 \lambda^2 \xi} \text{ Depairing critical current material specific} \]

\[ J_{C_p}^{\text{max}} \approx 3 \times 10^7 \text{ A/cm}^2 \]
Phase Diagram of a Type II Superconductor

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Please see: Phase Diagram from http://www.futurescience.com/manual/sc1000.html#C

Critical State Model of Bean and Livingston

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Superconducting Wire

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