Notation for Mean, Variance, and Correlation

- Consider random variables \( x \) and \( y \) with probability density functions \( f_x(x) \) and \( f_y(y) \) and joint probability function \( f_{xy}(x,y) \)
  - Expected value (mean) of \( x \) is
    \[
    \bar{x} = E(x) = \int_{-\infty}^{\infty} x f_x(x) \, dx
    \]
    - Note: we will often abuse notation and denote \( \bar{x} \) as a random variable (i.e., noise) rather than its mean
  - The variance of \( x \) (assuming it has zero mean) is
    \[
    \bar{x}^2 = E(x^* x) = \int_{-\infty}^{\infty} x^* x f_x(x) \, dx
    \]
  - A useful statistic is
    \[
    \bar{x} y = E(xy) = \int_{-\infty}^{\infty} xy f_{xy}(x, y) \, dxdy
    \]
    - If the above is zero, \( x \) and \( y \) are said to be uncorrelated
Relationship Between Variance and Spectral Density

- **Two-sided spectrum**
  \[
  \bar{x^2} = \int_{-f_2}^{-f_1} S_x(f) \, df + \int_{f_1}^{f_2} S_x(f) \, df
  \]
  - Since spectrum is symmetric
  \[\Rightarrow \bar{x^2} = 2 \int_{f_1}^{f_2} S_x(f) \, df\]

- **One-sided spectrum defined over positive frequencies**
  - Magnitude defined as twice that of its corresponding two-sided spectrum

- In the next few lectures, we assume a one-sided spectrum for all noise analysis
The Impact of Filtering on Spectral Density

- For the random signal passing through a linear, time-invariant system with transfer function $H(f)$

$$S_y(f) = |H(f)|^2 S_x(f)$$

- We see that if $x(t)$ is amplified by gain $A$, we have

$$S_y(f) = A^2 S_x(f) \quad \Rightarrow \quad \overline{y^2} = A^2 \overline{x^2}$$

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Noise in Resistors

- Can be described in terms of either voltage or current

\[
\overline{e_n^2} = 4kTR\Delta f
\]

- \( k \) is Boltzmann’s constant

\[
k = 1.38 \times 10^{-23} J/K
\]

- \( T \) is temperature (in Kelvins)
  - Usually assume room temperature of 27 degrees Celsius

\[
\Rightarrow T = 300K
\]
Noise In Inductors and Capacitors

- Ideal capacitors and inductors have no noise!

- In practice, however, they will have parasitic resistance
  - Induces noise
  - Parameterized by adding resistances in parallel/series with inductor/capacitor
    - Include parasitic resistor noise sources
Noise in CMOS Transistors (Assumed in Saturation)

Transistor Noise Sources
- Drain Noise (Thermal and 1/f)
- Gate Noise (Induced and Routing Parasitic)

- Modeling of noise in transistors must include several noise sources
  - Drain noise
    - Thermal and 1/f – influenced by transistor size and bias
  - Gate noise
    - Induced from channel – influenced by transistor size and bias
    - Caused by routing resistance to gate (including resistance of polysilicon gate)
      - Can be made negligible with proper layout such as fingering of devices
Drain Noise – Thermal (Assume Device in Saturation)

- Thermally agitated carriers in the channel cause a randomly varying current
  \[ \left. \frac{i_{nd}^2}{\Delta f} \right|_{th} = 4kT \gamma g_{do} \Delta f \]

  - \( \gamma \) is called excess noise factor
    - = 2/3 in long channel
    - = 2 to 3 (or higher!) in short channel NMOS (less in PMOS)

  - \( g_{do} \) will be discussed shortly  
    (Note: \( g_{do} = \frac{g_m}{\alpha} \))
Drain Noise – 1/f (Assume Device in Saturation)

- Traps at channel/oxide interface randomly capture/release carriers
  \[ \langle i_{nd}^2 \rangle_{1/f} = \frac{K_f}{f^n} \Delta f \approx \frac{K}{f} g_m^2 \frac{W L C_{ox}^2}{\Delta f} \]
  - Parameterized by \( K_f \) and \( n \)
    - Provided by fab (note \( n \approx 1 \))
  - Currently: \( K_f \) of PMOS << \( K_f \) of NMOS due to buried channel
- To minimize: want large area (high WL)
**Induced Gate Noise (Assume Device in Saturation)**

- Fluctuating channel potential couples capacitively into the gate terminal, causing a noise gate current

\[
\overline{i_{ng}^2} = 4kT\delta g_{do} \left( \frac{2\pi f}{\sqrt{5}/\alpha(g_m/C_{gs})} \right)^2 \Delta f \quad 4kT\delta g_{do}
\]

- \( \delta \) is gate noise coefficient
  - Typically assumed to be \( 2\gamma \)
  - Correlated to drain noise!

(Note: \( \alpha = g_m/g_{do} \))

---

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Useful References on MOSFET Noise

- **Thermal Noise**

- **Gate Noise**
  - Jung-Suk Goo et. al., “The Equivalence of van der Ziel and BSIM4 Models in Modeling the Induced Gate Noise of MOSFETS”, IEDM 2000, 35.2.1-35.2.4
**Drain-Source Conductance: $g_{do}$**

- $g_{do}$ is defined as channel resistance with $V_{ds}=0$
  - Transistor in triode, so that

$$I_d = \mu_n C_{ox} \frac{W}{L} \left((V_{gs} - V_T)V_{ds} - \frac{V_{ds}^2}{2}\right)$$

$$\Rightarrow g_{do} = \left. \frac{dI_d}{dV_{ds}} \right|_{V_{ds}=0} = \mu_n C_{ox} \frac{W}{L} (V_{gs} - V_T)$$

- Equals $g_m$ for long channel devices
- Key parameters for $0.18\mu$ NMOS devices

$$\mu_n = 327.4 \text{ cm}^2/(V \cdot \text{s})$$
$$t_{ox} = 4.1 \times 10^{-9} \text{ m} \quad \epsilon_o = 3.9(8.85 \times 10^{-12}) \text{ F/m}$$

$$\Rightarrow \mu_n C_{ox} = \mu_n \frac{\epsilon_o}{t_{ox}} = 275.6 \times 10^{-6} \text{ F}/(V \cdot \text{s})$$

$$V_T = 0.48 \text{ V}$$
Plot of \( g_m \) and \( g_{do} \) versus \( V_{gs} \) for 0.18\( \mu \) NMOS Device

For \( V_{gs} \) bias voltages around 1.2 V: \[ \alpha = \frac{g_m}{g_{do}} \approx \frac{1}{2} \]

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**Plot of \(g_m\) and \(g_{do}\) versus \(I_{dens}\) for 0.18\(\mu\) NMOS Device**

Transconductances \(g_m\) and \(g_{do}\) versus Current Density

Transconductance \(g_m\) (simulated in Hspice)

\[g_{do} = \mu_n C_{ox} W/L (V_{gs} - V_T)\]

\[W/L = \frac{1.8\mu}{0.18\mu}\]

\(V_{gs}\), \(I_d\), \(M_1\), \(W\), \(L\)
Noise Sources in a CMOS Amplifier

\( e_{nG}, e_{nD}, e_{ndeg} \): noise sources of external resistors

\( R_{gpar}, e_{ngpar} \): parasitic gate resistance and its noise

\( \bar{i}_{ng} \): induced gate noise,

\( g_g \): caused by distributed nature of channel

\[ g_g = \frac{w^2 C_{gs}^2}{5 g_{d0}} \]

\( i_{nd} \): drain noise (thermal and 1/f)

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Remove Model Components for Simplicity

$R_{gpar}$, $\overline{e_{ngpar}}$: can make negligible with proper layout

$g_g$: assume to be negligible (for $w \ll w_t$)

$C_{sb}$, $C_{gd}$, $C_{db}$, $g_{mb}$: too painful to include for calculations

$r_o$: impact is minor since $R_D$ is small (for high bandwidth)

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**Key Noise Sources for Noise Analysis**

![Diagram of noise sources in a transistor circuit]

- **Transistor gate noise**
  \[
  \langle i_{ng}^2 \rangle = 4kT \delta g_g \Delta f
  \]
  
  where \( g_g = \frac{w^2 C_{gs}^2}{5 g_{d0}} \)

- **Transistor drain noise**
  \[
  \langle i_{nd}^2 \rangle = 4kT \gamma g_{do} \Delta f + \frac{K_f}{f_n} \Delta f
  \]
  
  Thermal noise \quad 1/f noise
Apply Thevenin Techniques to Simplify Noise Analysis

Assumption: noise independent of load resistor on drain

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Calculation of Equivalent Output Noise for Each Case

\[ i_{\text{out}} = g_m Z_{gs} \bar{i}_{ng} + \eta \bar{i}_{nd} \]

\[ \bar{i}_{ndg} = g_m Z_{gs} \bar{i}_{ng} + \eta \bar{i}_{nd} \]
Calculation of $Z_{gs}$

- Write KCL equations

\begin{align}
(1) \quad -i_{test} + \frac{v_{test}}{1/(sC_{gs})} + g_{m}v_{test} &= \frac{v_{1}}{Z_{\text{deg}}} \\
(2) \quad \frac{v_{test}}{Z_{g}} + \frac{v_{1}}{Z_{\text{deg}}} &= g_{m}v_{test}
\end{align}

- After much algebra:

\[ Z_{gs} = \frac{v_{test}}{i_{test}} = \frac{1}{sC_{gs}} \left| \frac{Z_{\text{deg}} + Z_{g}}{1 + g_{m}Z_{\text{deg}}} \right| \]
Calculation of $\eta$

- Determine $V_{gs}$ to find $i_{out}$ in terms of $i_{test}$
  
  \begin{align*}
  (1) \quad i_{out} &= i_{test} + g_m v_{gs} \\
  (2) \quad v_{gs} &= -v_1 \frac{1}{1/(sC_{gs}) + Z_g} \\
  (3) \quad v_1 &= i_{out} \left( Z_{deg} \parallel \left( \frac{1}{sC_{gs}} + Z_g \right) \right)
  \end{align*}

- After much algebra:

$$
\eta = \frac{i_{out}}{i_{test}} = 1 - \left( \frac{g_m Z_{deg}}{Z_{deg} + Z_g} \right) Z_{gs}
$$
Calculation of Output Current Noise Variance (Power)

To find noise variance:

$$i_{out} = i_{ndg} = \eta i_{nd} + g_m Z_{gs} i_{ng}$$

$$\overline{i_{ndg}^2} = \overline{i_{ndg}^* i_{ndg}} = (\eta^* i_{nd}^* + g_m Z_{gs}^* i_{ng}^*) (\eta i_{nd} + g_m Z_{gs} i_{ng})$$
Variance (i.e., Power) Calc. for Output Current Noise

- Noise variance calculation

\[
i_{ndg}^2 = |\eta|^2 i_{nd}^2 i_{nd}^* + i_{nd}^* i_{ng}^* g_m \eta^* Z_{gs} + i_{nd}^* (g_m \eta Z_{gs})^* + i_{ng}^* |g_m Z_{gs}|^2
\]

\[
= |\eta|^2 i_{nd}^2 + 2 \text{Re} \left\{ i_{nd}^* i_{ng}^* g_m \eta^* Z_{gs} \right\} + i_{ng}^* |g_m Z_{gs}|^2
\]

\[
= |\eta|^2 i_{nd}^2 + 2 \text{Re} \left\{ \frac{i_{nd}^* i_{ng}^*}{\sqrt{i_{nd}^2 i_{ng}^2}} \sqrt{i_{nd}^2 i_{ng}^2} g_m \eta^* Z_{gs} \right\} + i_{ng}^* |g_m Z_{gs}|^2
\]

- Define correlation coefficient \( c \) between \( i_{ng} \) and \( i_{nd} \)

\[
c = \frac{i_{nd}^* i_{ng}^*}{\sqrt{i_{nd}^2 i_{ng}^2}} \Rightarrow i_{ndg}^2 = |\eta|^2 i_{nd}^2 + 2 \text{Re} \left\{ c \sqrt{i_{nd}^2 i_{ng}^2} g_m \eta^* Z_{gs} \right\} + i_{ng}^* |g_m Z_{gs}|^2
\]
Parameterized Expression for Output Noise Variance

- Key equation from last slide
  \[
  \bar{i}_{nd g}^2 = \bar{i}_{nd}^2 \left( |\eta|^2 + 2 \text{Re} \left\{ c \sqrt{\frac{i_{ng}^2}{i_{nd}^2}} g_m \eta^* Z_{gs} \right\} + \frac{i_{ng}^2}{i_{nd}^2} g_m^2 |Z_{gs}|^2 \right)
  \]

- Solve for noise ratio
  \[
  \sqrt{\frac{i_{ng}^2}{i_{nd}^2}} g_m = g_m \sqrt{\frac{4kT \delta (wC_{gs})^2/(5g_{do})}{4kT \gamma g_{do}}} = \frac{g_m}{g_{do}} \sqrt{\frac{\delta}{5\gamma}} (wC_{gs})
  \]

- Define parameters $Z_{gsw}$ and $\chi_d$
  \[
  Z_{gsw} = wC_{gs} Z_{gs}, \quad \chi_d = \frac{g_m}{g_{do}} \sqrt{\frac{\delta}{5\gamma}}
  \]

  \[
  \Rightarrow \bar{i}_{nd g}^2 = \bar{i}_{nd}^2 \left( |\eta|^2 + 2 \text{Re} \left\{ c \chi_d \eta^* Z_{gsw} \right\} + \chi_d^2 |Z_{gsw}|^2 \right)
  \]
Small Signal Model for Noise Calculations

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( |\eta|^2 + 2 \text{Re} \{c\chi_d \eta^* Z_{gsw} \} + \chi_d^2 |Z_{gsw}|^2 \right)
\]

where:
\[
\frac{i_{nd}^2}{\Delta f} = 4kT\gamma g_{do}, \quad \chi_d = \frac{g_m}{g_{do}} \sqrt{\frac{\delta}{5\gamma}}, \quad Z_{gsw} = wC_{gs}Z_{gs}
\]

\[
Z_{gs} = \frac{1}{sC_{gs}} \left| \frac{Z_{deg} + Z_g}{1 + g_m Z_{deg}} \right| Z_{gs}
\]

\[
\eta = 1 - \left( \frac{g_m Z_{deg}}{Z_{deg} + Z_g} \right) Z_{gs}
\]
**Example:** Output Current Noise with $Z_s = R_s$, $Z_{deg} = 0$

- **Step 1:** Determine key noise parameters
  - For 0.18µ CMOS, we will assume the following
    \[
    c = -j0.55, \quad \gamma = 3, \quad \delta = 2\gamma = 6, \quad \frac{g_m}{g_{do}} = \frac{1}{2} \Rightarrow \chi_d = 0.32
    \]

- **Step 2:** calculate $\eta$ and $Z_{gs\text{sw}}$
  \[
  \eta = 1, \quad Z_{gs\text{sw}} = wC_{gs} \left( R_s \frac{1}{1 + jwC_{gs}R_s} \right) = \frac{wC_{gs}R_s}{1 + jwC_{gs}R_s}
  \]
**Calculation of Output Current Noise (continued)**

- **Step 3:** Plug values into the previously derived expression

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( 1 + 2 \text{Re} \left\{ -j |c| \chi_d Z_{gs} \right\} + \chi_d^2 |Z_{gs}^2| \right)
\]

**Drain Noise Multiplying Factor**

\[
Z_{gs} = wC_{gs} \left( R_s \frac{1}{j \omega C_{gs}} \right) = wC_{gs} R_s \frac{1}{1 + j \omega C_{gs} R_s}
\]

- For \( w \ll 1/(R_s C_{gs}) \):

\[
Z_{gs} \approx wC_{gs} R_s \quad \Rightarrow \quad \frac{i_{ndg}^2}{\Delta f} \approx \frac{i_{nd}^2}{\Delta f} \left( 1 + \chi_d^2 (wC_{gs} R_s)^2 \right)
\]

**Gate noise contribution**
Calculation of Output Current Noise (continued)

- **Step 3:** Plug values into the previously derived expression

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( 1 + 2 \Re \{-j|c|\chi_d Z_{gsw}\} + \chi_d^2 |Z_{gsw}|^2 \right)
\]

Drain Noise Multiplying Factor

\[
Z_{gsw} = wC_{gs} \left( R_s \| \frac{1}{jwC_{gs}} \right) = \frac{wC_{gs}R_s}{1 + jwC_{gs}R_s}
\]

- For \( w \gg 1/(R_s C_{gs}) \):

\[
Z_{gsw} \approx \frac{1}{j} \quad \Rightarrow \quad \frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left( 1 - 2|c|\chi_d + \chi_d^2 \right)
\]

Gate noise contribution
**Plot of Drain Noise Multiplying Factor (0.18µ NMOS)**

**Drain Noise Multiplying Factor Versus Frequency for 0.18µ NMOS Device**

- **f << 1/(2πR_s C_{gs})**
- **f >> 1/(2πR_s C_{gs})**

- **Conclusion: gate noise has little effect on common source amp when source impedance is purely resistive!**

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**Broadband Amplifier Design Considerations for Noise**

- **Drain thermal noise** is the chief issue of concern when designing amplifiers with > 1 GHz bandwidth
  - 1/f noise corner is usually less than 1 MHz
  - Gate noise contribution only has influence at high frequencies
- **Noise performance specification** is usually given in terms of input referred voltage noise
Here we focus on a narrowband of operation
- Don’t care about noise outside that band since it will be filtered out

Gate noise is a significant issue here
- Using reactive elements in the source dramatically impacts the influence of gate noise

Specification usually given in terms of Noise Figure
The Impact of Gate Noise with $Z_s = R_s + sL_g$

- **Step 1:** Determine key noise parameters
  - For 0.18µ CMOS, again assume the following
  \[
  c = -j0.55, \quad \gamma = 3, \quad \delta = 2\gamma = 6, \quad \frac{g_m}{g_{do}} = \frac{1}{2} \quad \Rightarrow \quad \chi_d = 0.32
  \]

- **Step 2:** Note that $\eta = 1$, calculate $Z_{gsw}$

\[
Z_{gsw} = \omega C_{gs} \left( \frac{1}{(R_s + j\omega L_g)\|j\omega C_{gs}} \right) = \frac{\omega C_{gs}(R_s + j\omega L_g)}{1 - \omega^2 L_g C_{gs} + j\omega C_{gs} R_s}
\]

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Evaluate $Z_{gsw}$ At Resonance

- Set $L_g$ such that it resonates with $C_{gs}$ at the center frequency ($w_0$) of the narrow band of interest
  
  $$\Rightarrow \frac{1}{\sqrt{L_gC_{gs}}} = w_0$$
  
  Note: $Q = \frac{1}{w_0C_{gs}R_s} = \frac{w_oL_g}{R_s}$

- Calculate $Z_{gsw}$ at frequency $w_0$
  
  $$Z_{gsw} = \frac{w_oC_{gs}(R_s + jw_oL_g)}{1 - w_o^2L_gC_{gs} + jw_oC_{gs}R_s} = w_oC_{gs}(Q^2R_s - j\sqrt{L_g/C_{gs}})$$
  
  $$= Q - j$$
The Impact of Gate Noise with $Z_s = R_s + sL_g$ (Cont.)

- **Key noise expression derived earlier**
  \[
  \frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left(1 + 2\text{Re} \left\{ -j|c|\chi_d Z_{gs} \right\} + \chi_d^2 |Z_{gs}|^2 \right)
  \]

- **Substitute in for $Z_{gs}$**
  \[
  2\text{Re} \left\{ -j|c|\chi_d Z_{gs} \right\} = 2\text{Re} \left\{ -j|c|\chi_d (Q - j) \right\} = -2|c|\chi_d
  \]
  \[
  \chi_d^2 |Z_{gs}|^2 = \chi_d^2 |Q - j|^2 = \chi_d^2 (Q^2 + 1)
  \]
  \[
  \Rightarrow \quad \frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left(1 - 2|c|\chi_d + \chi_d^2 (Q^2 + 1) \right)
  \]
  *Gate noise contribution*

- **Gate noise contribution is a function of $Q$!**
  - Rises monotonically with $Q$
At What Value of Q Does Gate Noise Exceed Drain Noise?

- Determine crossover point for Q value

\[
\frac{i_{ndg}^2}{\Delta f} = \frac{i_{nd}^2}{\Delta f} \left(1 - 2|c|\chi_d + \chi_d^2(Q^2 + 1)\right) = \frac{i_{nd}^2}{\Delta f} \quad 1
\]

\[
\Rightarrow Q = \sqrt{\frac{1}{\chi_d^2} - 1 + 2|c|/\chi_d} \quad (= 3.5 \text{ for } 0.18\mu \text{ specs})
\]

- Critical Q value for crossover is primarily set by process

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Calculation of the Signal Spectrum at the Output

- First calculate relationship between $v_{in}$ and $i_{out}$

$$i_{out, sig} = g_m v_{gs} = g_m \frac{1}{1 - \omega^2 L_g C_{gs} + j \omega R_s C_{gs}} V_{in}$$

- At resonance:

$$i_{out, sig} = g_m v_{gs} = g_m \frac{1}{j \omega_0 R_s C_{gs}} v_{in} = g_m (-jQ) v_{in}$$

- Spectral density of signal at output at resonant frequency

$$S_{i_{out, sig}}(f) = |g_m (-jQ)|^2 S_{i_{in}}(f) = (g_m Q)^2 S_{i_{in}}(f)$$

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Impact of Q on SNR (Ignoring $R_s$ Noise)

- **SNR (assume constant spectra, ignore noise from $R_s$)**:
  \[
  \text{SNR}_{out} = \frac{S_{\text{out, sig}}(f)}{S_{\text{out, noise}}(f)} \approx \frac{(g_m Q)^2 S_{\text{in}}(f)}{\frac{i_{\text{ng}}^2}{\Delta f}}
  \]

- **For small Q such that gate noise < drain noise**
  - $\text{SNR}_{out}$ improves dramatically as Q is increased

- **For large Q such that gate noise > drain noise**
  - $\text{SNR}_{out}$ improves very little as Q is increased
Noise Factor and Noise Figure

- **Definitions**

Noise Factor \( F = \frac{SNR_{in}}{SNR_{out}} \)

Noise Figure \( = 10 \log(\text{Noise Factor}) \)

- **Calculation of** \( SNR_{in} \) **and** \( SNR_{out} \)

\[
SNR_{in} = \frac{|\alpha|^2 v_{in}^2}{e_{nRs}^2} = \frac{v_{in}^2}{e_{nRs}^2} \quad \text{where} \quad \alpha = \frac{Z_{in}}{R_s + Z_{in}}
\]

\[
SNR_{out} = \frac{|\alpha|^2 |G_m|^2 v_{in}^2}{|\alpha|^2 |G_m|^2 e_{nRs}^2 + i_{nout}^2} \quad \text{where} \quad G_m = \frac{i_{out}}{v_x}
\]
Calculate Noise Factor (Part 1)

- First calculate SNR_{out} (must include R_s noise for this)
  - R_s noise calculation (same as for V_{in})
    \[ i_{out,R_s} = g_m(-jQ)\overline{e_{ns}} \Rightarrow S_{i_{out,R_s}}(f) = (g_mQ)^2 4kTR_s \]
  - SNR_{out}:
    \[ SNR_{out} = \frac{(g_mQ)^2 S_{i_{in}}(f)}{i_{ndg}^2/\Delta f + (g_mQ)^2 4kTR_s} \]

- Then calculate SNR_{in}:
  \[ SNR_{in} = \frac{S_{i_{in}}(f)}{e_{ns}^2/\Delta f} = \frac{S_{i_{in}}(f)}{4kTR_s} \]
Calculate Noise Factor (Part 2)

\[ SNR_{out} = \frac{|g_m Q|^2 S_{in}(f)}{i_{ndg}^2 / \Delta f + (g_m Q)^2 4kT R_s} \quad SNR_{in} = \frac{S_{in}(f)}{e_{ns}^2 / \Delta f} = \frac{S_{in}(f)}{4kT R_s} \]

- **Noise Factor calculation:**

\[
\text{Noise Factor} = \frac{SNR_{in}}{SNR_{out}} = \frac{i_{ndg}^2 / \Delta f + |g_m Q|^2 4kT R_s}{(g_m Q)^2 4kT R_s} \\
= 1 + \frac{i_{ndg}^2 / \Delta f}{(g_m Q)^2 4kT R_s}
\]

- **From previous analysis**

\[
\frac{i_{ndg}^2}{\Delta f} = 4kT \gamma_{do} \left( 1 - 2|c| \chi_d + (Q^2 + 1) \chi_d^2 \right)
\]

\[ \Rightarrow \text{Noise Factor} = 1 + \frac{\gamma_{do} \left( 1 - 2|c| \chi_d + (Q^2 + 1) \chi_d^2 \right)}{(g_m Q)^2 R_s} \]
Calculate Noise Factor (Part 3)

\[
\text{Noise Factor} = 1 + \frac{\gamma g_{do} \left(1 - 2|c|\chi_d + (Q^2 + 1)\chi_d^2\right)}{(g_m Q)^2 R_s}
\]

- Modify denominator using expressions for \(Q\) and \(w_t\)

\[
Q = \frac{1}{w_o R_s C_{gs}}, \quad w_t \approx \frac{g_m}{C_{gs}}
\]

\[
\Rightarrow (g_m Q)^2 R_s = g_m^2 Q \frac{R_s}{w_o R_s C_{gs}} = g_m Q \frac{g_m}{C_{gs} w_o} \frac{1}{w_o} = g_m Q \frac{w_t}{w_o}
\]

- Resulting expression for noise factor:

\[
\text{Noise Factor} = 1 + \left(\frac{w_o}{w_t}\right) \gamma \left(\frac{g_{do}}{g_m}\right) \frac{1}{Q} \left(1 - 2|c|\chi_d + (Q^2 + 1)\chi_d^2\right)
\]

Noise Factor scaling coefficient

- Noise factor primarily depends on \(Q\), \(w_o/w_t\), and process specs

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Minimum Noise Factor

\[
\text{Noise Factor} = 1 + \left( \frac{w_o}{w_t} \right) \gamma \left( \frac{g_{do}}{g_m} \right) \frac{1}{Q} \left( 1 - 2|c| \chi_d + (Q^2 + 1) \chi_d^2 \right)
\]

Noise Factor scaling coefficient

- We see that the noise factor will be minimized for some value of \( Q \)
  - Could solve analytically by differentiating with respect to \( Q \) and solving for peak value (i.e. where deriv. = 0)
- In Tom Lee’s book (pp 272-277), the minimum noise factor for the MOS common source amplifier (i.e. no degeneration) is found to be:

\[
\text{Min Noise Factor} = 1 + \left( \frac{w_o}{w_t} \right) \frac{2}{\sqrt{5}} \sqrt{\gamma \delta (1 - |c|^2)}
\]

Noise Factor scaling coefficient

- How do these compare?
Plot of Minimum Noise Factor and Noise Factor Vs. Q

Noise Factor Scaling Coefficient Versus Q for 0.18µ NMOS Device

Achievable values as a function of Q under the constraint that

\[
\frac{1}{\sqrt{L_g C_{gs}}} = w_o
\]

Minimum across all values of Q and

\[
\frac{1}{\sqrt{L_g C_{gs}}} \approx Q^2
\]

Note: curves meet if we approximate \(Q^2 + 1 \approx Q^2\) for

- \(c = -j0\)
- \(c = -j0.55\)
- \(c = -j1\)
Achieving Minimum Noise Factor

- For common source amplifier without degeneration
  - Minimum noise factor can only be achieved at resonance if gate noise is uncorrelated to drain noise (i.e., if $c = 0$) – we’ll see this next lecture
  - We typically must operate slightly away from resonance in practice to achieve minimum noise factor since $c$ will be nonzero

- How do we determine the optimum source impedance to minimize noise figure in classical analysis?
  - Next lecture!