What Happens When the Wave Hits a Boundary?

- Reflections can occur

![Diagram showing incident and reflected waves](image-url)
What Happens When the Wave Hits a Boundary?

- At boundary
  - Orientation of H-field flips with respect to E-field
  - Current reverses direction with respect to voltage

![Diagram showing incident and reflected waves](image-url)
What Happens At The Load Location?

- Voltage and currents at load are ratioed according to the load impedance.

\[
\frac{V_i + V_r}{I_i - I_r} = Z_L
\]
**Relate to Characteristic Impedance**

- From previous slide

\[
\frac{V_i + V_r}{I_i - I_r} = \frac{V_i}{I_i} \left( \frac{1 + V_r/V_i}{1 - V_r/I_i} \right) = Z_L
\]

- Voltage and current ratio in transmission line set by it characteristic impedance

\[
\frac{V_i}{I_i} = \frac{V_r}{I_r} = Z_o \quad \Rightarrow \quad \frac{I_r}{I_i} = \frac{V_r}{V_i}
\]

- Substituting:

\[
Z_o \left( \frac{1 + V_r/V_i}{1 - V_r/V_i} \right) = Z_L
\]
Define Reflection Coefficient

- Definition: 
  \[ \Gamma_L = \frac{V_r}{V_i} \]
  - No reflection if \( \Gamma_L = 0 \)

- Relation to load and characteristic impedances
  \[ Z_o \left( \frac{1 + \Gamma_L}{1 - \Gamma_L} \right) = Z_L \]

- Alternate expression
  \[ \Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \]
  - No reflection if \( Z_L = Z_o \)
Parameterization of High Speed Circuits/Passives

- Circuits or passive structures are often connected to transmission lines at high frequencies
  - How do you describe their behavior?
Calculate Response to Input Voltage Sources

- Assume source impedances match their respective transmission lines

\[
\begin{align*}
\text{Same value} & \quad \text{by definition} \\
Z_1 & \quad \text{Vin1} \\
& \quad \text{Transmission Line 1} \\
\end{align*}
\]
Calculate Response to Input Voltage Sources

- Sources create incident waves on their respective transmission line
- Circuit/passive network causes
  - Reflections on same transmission line
  - Feedthrough to other transmission line
Calculate Response to Input Voltage Sources

- Reflections on same transmission line are parameterized by $\Gamma_L$
  - Note that $\Gamma_L$ is generally different on each side of the circuit/passive network

How do we parameterize feedthrough to the other transmission line?
S-Parameters – Definition

- Model circuit/passive network using 2-port techniques
  - Similar idea to Thevenin/Norton modeling

- Defining equations:

\[
\begin{align*}
\frac{V_{r1}}{\sqrt{Z_1}} &= S_{11} \frac{V_{i1}}{\sqrt{Z_1}} + S_{12} \frac{V_{i2}}{\sqrt{Z_2}} \\
\frac{V_{r2}}{\sqrt{Z_2}} &= S_{21} \frac{V_{i1}}{\sqrt{Z_1}} + S_{22} \frac{V_{i2}}{\sqrt{Z_2}}
\end{align*}
\]
\[
\frac{V_{r1}}{\sqrt{Z_1}} = S_{11} \frac{V_{i1}}{\sqrt{Z_1}} + S_{12} \frac{V_{i2}}{\sqrt{Z_2}} \quad \text{and} \quad \frac{V_{r2}}{\sqrt{Z_2}} = S_{21} \frac{V_{i1}}{\sqrt{Z_1}} + S_{22} \frac{V_{i2}}{\sqrt{Z_2}}
\]

**Set** \( V_{in2} = 0 \)

\[
\Rightarrow S_{11} = \frac{V_{r1}}{V_{i1}} = \Gamma_{L1}
\]

\[
\Rightarrow S_{21} = \sqrt{\frac{Z_1}{Z_2}} \left( \frac{V_{r2}}{V_{i1}} \right)
\]

**Set** \( V_{in1} = 0 \)

\[
\Rightarrow S_{22} = \frac{V_{r2}}{V_{i2}} = \Gamma_{L2}
\]

\[
\Rightarrow S_{12} = \sqrt{\frac{Z_2}{Z_1}} \left( \frac{V_{r1}}{V_{i2}} \right)
\]
Note: Alternate Form for $S_{21}$ and $S_{12}$

\[ V_{i1} = \frac{V_{in1}}{2} \]

\[ V_{i2} = \frac{V_{in2}}{2} \]

\[ \Gamma_{L1} \]

\[ \Gamma_{L2} \]

\[ \begin{align*}
    Z_1 &\quad V_{r1} \\
    V_{in1} &\quad \Gamma_{L1} \\
    \Gamma_{L1} &\quad V_{i1} \\
    \Gamma_{L2} &\quad V_{i2} \\
    Z_2 &\quad V_{r2} \\
    V_{in2} &\end{align*} \]

\[ \Rightarrow S_{11} = \frac{V_{r1}}{V_{i1}} = \Gamma_{L1} \]

\[ \Rightarrow S_{21} = 2 \sqrt{\frac{Z_1}{Z_2}} \left( \frac{V_{r2}}{V_{in1}} \right) \]

\[ \Rightarrow S_{12} = 2 \sqrt{\frac{Z_2}{Z_1}} \left( \frac{V_{r1}}{V_{in2}} \right) \]

\[ \text{set } V_{in2} = 0 \]

\[ \Rightarrow S_{11} = \frac{V_{r1}}{V_{i1}} = \Gamma_{L1} \]

\[ \Rightarrow S_{21} = 2 \sqrt{\frac{Z_1}{Z_2}} \left( \frac{V_{r2}}{V_{in1}} \right) \]

\[ \text{set } V_{in1} = 0 \]

\[ \Rightarrow S_{12} = 2 \sqrt{\frac{Z_2}{Z_1}} \left( \frac{V_{r1}}{V_{in2}} \right) \]
Key issue – two-port is parameterized with respect to the left and right side load impedances ($Z_1$ and $Z_2$)
- Need to recalculate $S_{11}$, $S_{21}$, etc. if $Z_1$ or $Z_2$ changes
- Typical assumption is that $Z_1 = Z_2 = 50$ Ohms
Macro-modeling for Distributed, Linear Networks

Key parameters for a transmission line
- Characteristic impedance (only impacts S-parameter calculations)
- Delay (function of length and $\mu$, $\varepsilon$)
- Loss (ignore for now)

Key parameters for circuits/passives
- S-parameters

We would like an overall macro-model for simulation
Macro-modeling for Distributed, Linear Networks

- Model transmission line as a delay element
  - If lossy, could also add an attenuation factor (which is a function of its length)
- Model circuits/passives with S-parameter 2-ports
- Model source and load with custom blocks
Macro-modeling for Distributed, Linear Networks

\[ V_s \rightarrow Z_s \rightarrow Z_1 \rightarrow Z_2 \rightarrow Z_3 \rightarrow V_{out} \]

- \( Z_1 \): Length = \( d_1 \)
- \( Z_2 \): Length = \( d_2 \)
- \( Z_3 \): Length = \( d_3 \)

\[ \text{delay}_1 = \frac{d_1}{\text{velocity}} = \sqrt{LCd_1} = \sqrt{\mu\varepsilon d_1} \]

\[ \text{delay}_2 = \sqrt{\mu\varepsilon d_2} \]

\[ \text{delay}_3 = \sqrt{\mu\varepsilon d_3} \]

\[ V_{out} = \frac{Z_L}{Z_L + Z_3} \]

- \( Z_s \):
  - \( Z_s + Z_1 \)
  - \( Z_s - Z_1 \)

- S-param. 2-port
  - \( V_{i1} \rightarrow V_{r1} \)
  - \( V_{i2} \rightarrow V_{r2} \)

- \( V_s \):
  - \( \frac{Z_1}{Z_1 + Z_s} \)
  - \( \frac{Z_s - Z_1}{Z_s + Z_1} \)

- \( Z_L = Z_1 \)
- \( Z_L = Z_2 \)
- \( Z_L = Z_3 \)

- \( V_{out} \):
  - \( \frac{Z_L - Z_3}{Z_L + Z_3} \)
Note for CppSim Simulations

- CppSim does block-by-block computation
  - Feedback introduces artificial delays in simulation
- Prevent artificial delays by
  - Ordering blocks according to input-to-output signal flow
  - Creating an additional signal in CppSim modules to pass previous sample values
  - Note: both are already done for you in Homework #1
**S-Parameter Calculations – Example 1**

- **Set \( V_{i2} = 0 \)**
  
  \[
  V_{r1} = \Gamma_1 V_{i1} = \frac{Z_2 - Z_1}{Z_2 + Z_1} V_{i1}
  \]
  
  \[
  V_{r2} = V_{i1} + V_{r1} = (1 + \Gamma_1) V_{i1}
  \]

- **Set \( V_{i1} = 0 \)**
  
  \[
  V_{r2} = \Gamma_2 V_{i2} = \frac{Z_1 - Z_2}{Z_1 + Z_2} V_{i2}
  \]
  
  \[
  V_{r1} = V_{i2} + V_{r2} = (1 + \Gamma_2) V_{i2}
  \]

\[
\Rightarrow \quad S_{11} = \Gamma_1
\]

\[
\Rightarrow \quad S_{21} = \sqrt{\frac{Z_1}{Z_2}} (1 + \Gamma_1)
\]

\[
\Rightarrow \quad S_{22} = \Gamma_2
\]

\[
\Rightarrow \quad S_{12} = \sqrt{\frac{Z_2}{Z_1}} (1 + \Gamma_2)
\]
S-Parameter Calculations – Example 2

- **Same as before:**

  \[
  S_{11} = \Gamma_1 \\
  S_{21} = \sqrt{\frac{Z_1}{Z_2}}(1 + \Gamma_1) \\
  S_{22} = \Gamma_2 \\
  S_{12} = \sqrt{\frac{Z_2}{Z_1}}(1 + \Gamma_2)
  \]

- **But now:**

  \[
  \Gamma_1 = \frac{Z_2 || (1/sC) - Z_1}{Z_2 || (1/sC) + Z_1} \\
  \Gamma_2 = \frac{Z_1 || (1/sC') - Z_2}{Z_1 || (1/sC') + Z_2}
  \]
S-Parameter Calculations – Example 3

- The S-parameter calculations are now more involved
  - Network now has more than one node
- This is a homework problem
Impedance Transformers
**Matching for Voltage versus Power Transfer**

- Consider the voltage divider network

- For maximum voltage transfer

\[ R_L \to \infty \implies V_{out} \to V_s \]

- For maximum power transfer

\[ R_L = R_S \implies P_{out} = \frac{|V_{out}|^2}{R_L} = \frac{|V_s|^2}{4R_S} \]

Which one do we want?
Note: Maximum Power Transfer Derivation

- **Formulation**

  \[ P_{out} = I^2 R_L = \left( \frac{V_s}{R_S + R_L} \right)^2 R_L = \frac{R_L}{(R_S + R_L)^2} V_s^2 \]

- **Take the derivative and set it to zero**

  \[ \frac{dP_{out}}{dR_L} = R_L (-2)(R_S + R_L)^{-3} + (R_S + R_L)^{-2} = 0 \]

  \[ \Rightarrow 2R_L = R_S + R_L \quad \Rightarrow \quad R_L = R_S \]
Voltage Versus Power

- For most communication circuits, voltage (or current) is the key signal for detection
  - Phase information is important
  - Power is ambiguous with respect to phase information
    - Example:

- For high speed circuits with transmission lines, achieving maximum power transfer is important
  - Maximum power transfer coincides with having zero reflections (i.e., $\Gamma_L = 0$)

Can we ever win on both issues?
Broadband Impedance Transformers

- Consider placing an ideal transformer between source and load

- Transformer basics (passive, zero loss)
  1) \( V_{out} = NV_{in} \)  
  2) Power In = Power Out

  \[ V_{in}I_{in} = V_{out}I_{out} \]

  From (1) and (2): \( V_{in}I_{in} = NV_{in}I_{out} \)  \( \Rightarrow I_{out} = \frac{I_{in}}{N} \)

- Transformer input impedance

  \[ R_{in} = \frac{V_{in}}{I_{in}} = \frac{V_{out}/N}{NI_{out}} = \frac{1}{N^2}R_L \]
What Value of $N$ Maximizes Voltage Transfer?

- Derive formula for $V_{out}$ versus $V_{in}$ for given $N$ value

\[
V_{out} = NV_{in} = N \frac{R_{in}}{R_s + R_{in}} V_s = N \frac{R_L/N^2}{R_s + R_L/N^2} V_s
\]

\[
= N \frac{R_L}{R_L + N^2 R_s} V_s
\]

- Take the derivative and set it to zero

\[
\frac{dV_{out}}{dN} = NR_L(-1)(R_L+N^2 R_s)^{-2}2NR_s + R_L(R_L+N^2 R_s)^{-1} = 0
\]

\[
\Rightarrow -2N^2 R_s (R_L+N^2 R_s)^{-2} + (R_L+N^2 R_s)^{-1} = 0
\]

\[
\Rightarrow -2N^2 R_s = R_L + N^2 R_s \quad \Rightarrow \quad N^2 = \frac{R_L}{R_s}
\]
What is the Input Impedance for Max Voltage Transfer?

- We know from basic transformer theory that input impedance into transformer is
  \[ R_{in} = \frac{1}{N^2} R_L \]

- We just learned that, to maximize voltage transfer, we must set the transformer turns ratio to
  \[ N^2 = \frac{R_L}{R_s} \]

- Put them together
  \[ R_{in} = \frac{1}{N^2} R_L = \frac{1}{R_L/R_s} R_L = R_s \]

So, \( N \) should be set for max power transfer into transformer to achieve the maximum voltage transfer at the load!
**Benefit of Impedance Matching with Transformers**

- Transformers allow maximum voltage and power transfer relationship to coincide

- Turns ratio for max power/voltage transfer

\[ N^2 = \frac{R_L}{R_s} \]

- Resulting voltage gain (can exceed one!)

\[ V_{out} = NV_{in} = N \left( \frac{1}{2} V_s \right) = \sqrt{\frac{R_L}{R_s}} \left( \frac{1}{2} V_s \right) \]
The Catch

- It’s hard to realize a transformer with good performance over a wide frequency range
  - Magnetic materials have limited frequency response
  - Inductors have self-resonant frequencies, losses, and mediocre coupling to other inductors without magnetic material

- For wireless applications, we only need transformers that operate over a small frequency range
  - Can we take advantage of this?
Consider Resonant Circuits (Chap. 4 of Lee’s Book)

Series Resonant Circuit

\[ Z_{in} = \frac{1}{j\omega C_s} + j\omega L_s + R_s \]

\[ = R_s \quad \text{for} \quad w = \frac{1}{\sqrt{L_s C_s}} = w_0 \]

\[ Q = \frac{w_0 L_s}{R_s} = \frac{1}{w_0 C_s R_s} \]

Parallel Resonant Circuit

\[ Z_{in} = \frac{1}{j\omega C_p || j\omega L_p || R_p} \]

\[ = R_p \quad \text{for} \quad w = \frac{1}{\sqrt{L_p C_p}} = w_0 \]

\[ Q = \frac{R_p}{w_0 L_p} = w_0 C_p R_p \]

Key insight: resonance allows \( Z_{in} \) to be purely real despite the presence of reactive elements
Comparison of Series and Parallel RL Circuits

<table>
<thead>
<tr>
<th>Series RL Circuit</th>
<th>Parallel RL Circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{in} = j \omega_0 L_s + R_s$</td>
<td>$Z_{in} = j \omega_0 L_p</td>
</tr>
</tbody>
</table>

- Equate real and imaginary parts of the left and right expressions (so that $Z_{in}$ is the same for both)
  - Also equate Q values

\[
R_p = R_s (Q^2 + 1) \approx R_s Q^2 \quad \text{(for } Q \gg 1) \\
L_p = L_s \left( \frac{Q^2 + 1}{Q^2} \right) \approx L_s \quad \text{(for } Q \gg 1) 
\]
Comparison of Series and Parallel RC Circuits

**Series RC Circuit**

\[ Z_{in} = R_s + \frac{1}{j\omega C_s} \]

\[ Q = \frac{1}{\omega C_s R_s} \]

**Parallel RC Circuit**

\[ Z_{in} = \frac{1}{\frac{1}{R_p} + \frac{1}{j\omega C_p}} \]

\[ Q = \omega C_p R_p \]

- Equate real and imaginary parts of the left and right expressions (so that \(Z_{in}\) is the same for both)
- Also equate Q values

\[ R_p = R_s (Q^2 + 1) \approx R_s Q^2 \quad \text{(for } Q \gg 1) \]

\[ C_p = C_s \left( \frac{Q^2}{Q^2 + 1} \right) \approx C_s \quad \text{(for } Q \gg 1) \]
A Narrowband Transformer: The L Match

- Assume $Q \gg 1$

- So, at resonance

\[
Z_{in} = R_p \approx Q^2 R_s \quad \text{(purely real)}
\]

- Transformer steps up impedance!
Alternate Implementation of L Match

- Assume $Q >> 1$

- So, at resonance

\[ Z_{in} = R_s \approx \frac{R_p}{Q^2} \] (purely real)

- Transformer steps down impedance!
The $\pi$ Match

- Combines two L sections

- Provides an extra degree of freedom for choosing component values for a desired transformation ratio
The T Match

- Also combines two L sections

- Again, benefit is in providing an extra degree of freedom in choosing component values
Tapped Capacitor as a Transformer

To first order:

\[
\frac{R_{in}}{R_L} \approx \left( \frac{C_1 + C_2}{C_1} \right)^2
\]

- Useful in VCO design
- See Chapter 4 of Tom Lee’s book for analysis