QUICKS 1

SOLUTIONS
Problem 1

A. Draw normal $P(t)$ waveforms for the left ventricle, left atrium, and aorta. Show two complete cardiac cycles, and use typical normal values for the pressures. Use the time axis provided in Figure 1.1a, and assume a heart rate of 60 bpm.

\[
C.O. = \frac{\dot{V}_{O_2}}{C^A_{O_2} - C^V_{O_2}} = \frac{363 \text{ ml } O_2/\text{min}}{(200 - 14.5) \text{ ml } O_2/\text{liter blood}} = \frac{363}{55} = 6.6 \text{ L/min}
\]

So

\[
SV = \frac{6.6 \text{ L/min}}{60 \text{ beat/min}} = 110 \text{ cc/beat}
\]

(See Figure.)

B. The cardiac output was measured using the Fick method.

- Oxygen uptake: 363 ml O$_2$ per minute
- Arterial oxygen content: 200 ml O$_2$ per liter of blood
- Mixed venous oxygen content: 145 ml O$_2$ per liter of blood

Using this data together with your $P(t)$ waveforms, draw the corresponding P-V loop for the LV. Assume an end-diastolic LV volume of 170 cc., and a LV “dead” volume of 15 cc for both systole and diastole. Draw linear systolic and diastolic P-V curves, and use the axes provided.

See Figure.

C. Correlate the following landmarks on the P-V loop with the appropriate points on the $P(t)$ curves using the numeric labels below:

- a: begin LV contraction
- b: peak LV pressure
- c: begin LV filling
- d: end ejection
- e: begin LV ejection

See Figure.

D. “Ejection fraction” (EF) is defined as the percentage of the end-diastolic volume that is ejected during systole. What is the EF in this case? (Normal $>$ 55%.)

\[
EF = \frac{110}{170} = 64.7\%
\]
E. A papillary muscle in the LV ruptures. (Assume that there are no functioning controls, and that the system has reached a new steady state.) The new arterial BP (systolic, diastolic, and mean) drops to 60% of its original value.

New BP:

\[
\begin{align*}
120 \times 0.6 &= 72 \\
80 \times 0.6 &= 48 \\
110 \times 0.6 &= 66
\end{align*}
\]

(i) Sketch two cardiac cycles showing the new \( P(t) \) waveforms, using the axes supplied in Figure 1.1b. Pay particular attention to the new amplitudes of the LV and LA pressures. Assume no change in the left ventricular end-diastolic pressure and volume.

See sketches for \( P(t) \), P-V loop. Note high pressure in atrium at end-systole due to blood leaking past mitral valve.

(ii) Sketch the new P-V loop on the same axes as part (B) above. Estimate the new stroke volume.

\[
\text{New stroke volume} \approx 170 - 42 = 128 \text{ cc}
\]

(iii) What is the new ejection fraction (using the definition in part D)?

\[
\text{EF}' = \frac{128}{170} = 75.3\%
\]

(iv) Crudely approximate the stroke volume delivered to the aorta by making use of the Windkessel approximation.

The pulse pressure had decreased from

\[
120 - 80 = 40 \text{ mmHg}
\]

to

\[
72 - 48 = 24 \text{ mmHg (60% of prior)}
\]

So, if we assume \( SV \) is proportional to pulse pressure,

\[
SV' = 0.6 \times 110 \text{ cc} = 66 \text{ cc}
\]

\( \uparrow \) original \( SV \)

(v) What is the “forward ejection fraction” (the percentage of the end-diastolic LV volume that is ejected into the aorta)?

\[
\text{Forward EF} = \frac{66}{170} = 38.8\%
\]
(vi) As a result of the papillary muscle rupture, a murmur appears. Indicate its temporal location on the time axis provided in Figure 1.1.

*It is heard throughout systole as a regurgitant jet enters the atrium. See Figure.*

Figure 1.1:
Figure 1.2:
**Problem 2**

We have used the lumped parameter model of the cardiovascular system that is shown in Figure 2.1. The following relationship was derived to relate cardiac output to the various model parameters (in operating region I):

\[
\text{C.O.} = \frac{P_{ms} - P_{th} - (P_{PA}^0 - P_{th}) \frac{C_a}{C_D}}{R_v + R_a \frac{C_a}{C_a + C_v} + \frac{1}{\tau C_D}}
\]

Figure 2.1: Lumped Parameter Model

**Part 1**

Using this expression and/or graphical analysis explain the expected changes in: (a) cardiac output, (b) arterial blood pressure, and (c) pulse pressure that would result from the following interventions, assuming an uncontrolled CV system and a heart rate of 60 bpm.

A. Increasing the peripheral resistance, \( R_a \).

B. Decreasing total blood volume.

C. Increasing left ventricular contractility.

D. Decreasing arterial capacitance, \( C_a \), by a factor of two.

E. Increasing the intra-thoracic pressure by 10 mmHg, and \( P_{ms} \) by 8 mmHg by blowing into a balloon.

**Part 2**

For each intervention above, sketch the expected qualitative changes in the CO/VR curves using the graphs below.
A. Increasing the peripheral resistance, $R_a$.

\[ \text{ABP} = \text{CO} \times R_a. \]  
There will be a slight decrease in CO, a large increase in \( \text{ABP} \), and a change in the \( \tau \) of \( \text{ABP} \) decay, but little change (a slight decrease) in pulse pressure.
B. Decreasing total blood volume.

\[ P_{ms} \text{ drops and the VR curve shifts to the left. CO drops. ABP drops proportionately because } ABP = C.O. \times R. \text{ Since HR does not change, pulse pressure also drops proportionately.} \]
C. Increasing left ventricular contractility.

\[
\begin{align*}
\text{Cardiac Output and Venous Return} & \quad \text{(L/min.)} \\
15 & \quad 10 \\
5 & \quad 0
\end{align*}
\]

\[
\begin{align*}
-5 & \quad 0 & \quad 5 & \quad 10 \\
\text{Right Atrial Pressure (mmHg)} & \quad \text{Cardiac Output (L/min.)} & \quad \text{Venous Return (L/min.)}
\end{align*}
\]

\[
\begin{align*}
\text{Equilibrium point} & \quad \text{Venous Return (L/min.)}
\end{align*}
\]

\[CO \text{ is not a function of } C_{LV}^L, \text{ so there is no change in } CO, \text{ ABP, or PP.}\]
D. Decreasing arterial capacitance, $C_a$, by a factor of two.

There is a slight increase in CO from 5.44 L/min to 5.96 L/min.

$C_a = 2:$

\[
P_{ms} = \frac{V_T - V_0}{C_a - C_v} = \frac{4000 - 3200}{2 + 100} = \frac{800}{102} = 7.84
\]

\[
CO = \frac{7.84 - (-5) - [15 - (-5)] \times \frac{2}{20}}{0.05 + 1 \cdot \left(\frac{2}{102}\right) + \frac{1}{1 \times 20}} = \frac{7.84 + 5 - 2}{0.05 + 0.196 + 0.05} = \frac{10.84}{0.1196} = 90.63 \text{ cc/sec}
\]

$C_a = 1:$

\[
P_{ms} = \frac{800}{101} = 7.92
\]

\[
CO = \frac{7.92 + 5 - 2}{0.05 + 1 \cdot \left(\frac{2}{101}\right) + 0.05} = \frac{10.92}{0.1099} = 99.36 \text{ cc/sec} = 5.96 \text{ L/min}
\]

Mean ABP will rise proportionately. The pulse pressure, however, will double since

\[
\text{Pulse Pressure} \approx \frac{SV}{C_a}
\]
E. Increasing the intra-thoracic pressure by 10 mmHg, and $P_{ms}$ by 8 mmHg by blowing into a balloon.

*Original CO by equation:*

$$CO \approx \frac{(7.8 + 5) - (15 + 5)\frac{2}{20}}{.05 + .02 + .05} = \frac{12.8 - 2}{.12} = \boxed{5.4 \text{ L/min}}$$

*With new $P_{th}, P_{ms}$:*

$$CO' = \frac{(15.8 - 5) - (15 - 5)0.1}{.12} = \frac{10.8 - 1}{.12} = 81.67 \text{ cc/sec} = \boxed{4.90 \text{ L/min}}$$
Table 1: Glossary of Symbols and Nominal Value for Model Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Normal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔV</td>
<td>stroke volume</td>
<td>96 cc</td>
</tr>
<tr>
<td>f = \frac{1}{T}</td>
<td>heart rate</td>
<td>60/min. = 1/sec.</td>
</tr>
<tr>
<td>T = T_S + T_D</td>
<td>duration of heart cycle</td>
<td>1 sec.</td>
</tr>
<tr>
<td>T_S</td>
<td>duration of systole</td>
<td>.3 sec.</td>
</tr>
<tr>
<td>T_D</td>
<td>duration of diastole</td>
<td>.7 sec.</td>
</tr>
<tr>
<td>C_D^r</td>
<td>diastolic capacitance of RV</td>
<td>20 ml/mmHg</td>
</tr>
<tr>
<td>C_D^l</td>
<td>diastolic capacitance of LV</td>
<td>10 ml/mmHg</td>
</tr>
<tr>
<td>C_S^r</td>
<td>minimum systolic capacitance of RV</td>
<td>2 ml/mmHg</td>
</tr>
<tr>
<td>C_S^l</td>
<td>minimum systolic capacitance of LV</td>
<td>.4 ml/mmHg</td>
</tr>
<tr>
<td>V_{max}^r, V_{max}^l</td>
<td>“maximum” volumes, RV, LV</td>
<td>200 cc</td>
</tr>
<tr>
<td>V_T = V + V_0</td>
<td>total volume of blood in peripheral vasculature</td>
<td>4000 ml</td>
</tr>
<tr>
<td>V_0</td>
<td>volume needed to fill peripheral vasculature without increasing pressure</td>
<td>3200 ml</td>
</tr>
<tr>
<td>C_a</td>
<td>arterial capacitance</td>
<td>2 ml/mmHg</td>
</tr>
<tr>
<td>C_v</td>
<td>venous capacitance</td>
<td>100 ml/mmHg</td>
</tr>
<tr>
<td>R_a</td>
<td>arterial resistance</td>
<td>1 mlHg/(ml/sec)</td>
</tr>
<tr>
<td>R_v</td>
<td>resistance to venous return</td>
<td>.05 mmHg/(ml/sec)</td>
</tr>
<tr>
<td>P_{th}</td>
<td>mean intrathoracic pressure</td>
<td>-5 mmHg</td>
</tr>
<tr>
<td>P_A^0</td>
<td>pulmonary artery pressure (end-systolic) referenced to mean intrathoracic pressure</td>
<td>15 mmHg</td>
</tr>
<tr>
<td>P_{ms}</td>
<td>mean systemic filling pressure (see text)</td>
<td>7.8 mmHg</td>
</tr>
<tr>
<td>P_v</td>
<td>peripheral venous pressure</td>
<td>6.1 mmHg</td>
</tr>
</tbody>
</table>
sphere is depolarized before repolarization begins. Assume the heart to be in the center of the spherical torso, and that all the assumptions underlying the dipole ECG theory are valid.

A. Sketch the three orthogonal scalar waveforms $V_x(t)$, $V_y(t)$, and $V_z(t)$ as defined in Figure 3.4 for one depolarization sequence. Label the time axis in terms of the radius of the spherical heart, $a$, and the velocity of propagation, $v$. [Note: try to be as quantitative as possible, but partial credit will be given for a qualitative answer.]
Figure 3.4:
Symmetry leads to cancellation of the y and z components of the heart vector. So only $V_x$ is non-zero.

First, calculate the equivalent heart vector, $\vec{M}_0$. We know it is in the $x$-direction. Its magnitude is the projection on $i_x$ of the individual components.

Figure 3.5:

Let $h$ be the thickness of the shell. At the interface of depolarized and polarized tissue there is a circular boundary of radius $a \sin \theta$. An elemental area, $dA$, may be defined as

$$dA = ha \sin \theta d\alpha$$

where $\alpha$ is the angle of rotation around the $x$-axis. Let $\vec{m}$ be the elemental current dipole per unit area, and the dipole moment associated with $dA$ will be

$$\vec{m} = mha \sin \theta d\alpha$$

$\vec{m}$ makes an angle $\left(\frac{\pi}{2} - \theta\right)$ with the $x$-axis for all $\alpha$. The projection of $\vec{m}$ on $i_x$ will therefore be

$$m_x = mha \sin^2 \theta d\alpha$$

The total net $x$-projection at a given $\theta$ would be

$$M_x(t) = \int_0^{2\pi} mha \sin^2 \theta(t) d\alpha = 2\pi mha \sin^2 \theta$$
But

\[ \theta = \omega t = \frac{v}{a} t \]

So

\[ M_x(t) = 2\pi mha \sin^2 \left( \frac{vt}{a} \right) \]

It is plotted in the figure below.
\[ v_x(t) = A \sin^2 \left( \frac{\omega t}{2} \right) \]

\[ V_y(t) \]

\[ V_z(t) \]