**HST.584 / 22.561 Problem Set #1 Solutions**

Marking Scheme: Question 1 – 3.5 points, Question 2 – 3 points, Question 3 – 3.5 points

General Comments: Watch out for radians! Angles should be expressed in radians and γ adjusted between linear and angular units accordingly \( (\text{Hz vs rad/s}) \)

1-a) \( B_1 = \frac{\theta}{T \gamma} \) for \( \theta = \pi /2, T = 1 \text{ ms}, \gamma = 2\pi \times 43 \times 10^6 \text{ rad/s/T} \)

\[ B_1 = 5.81 \times 10^{-6} T \]

This is 6 orders of magnitude smaller than a typical \( B_0 \).

1-b) \( B_1 = \frac{\mu_0 NI}{2r} \) for a short solenoid.

Rearranging, we can calculate \( I = 0.462 \text{ A} \) to produce our desired field. To estimate power, we need to estimate resistance (only a resistive element causes power loss – can’t dissipate power through an inductance). Let’s pick copper \( (\rho = 1.56 \times 10^{-8} \Omega \text{ m}) \) with a circular cross-section of 2 mm (maybe a little small for the current we’re pushing, but good enough for an order of magnitude estimate).

\[ R = \frac{\rho L}{A} \]

where \( L \) is the total length of wire \( (4 \text{ turns} \times \text{circumference} = 5.03 \text{ m}) \)

\[ R = 0.025 \Omega \]

\[ P = I^2 R = 5.33 mW \] for an estimate

1-c) \( B_{\text{eff}} \) is the vector sum of \( B_1 \) and our off-resonance contribution.

Off resonance \( = B_0 - \omega_{\text{rot}} / \gamma = 0.1163 mT \).

So \( B_{\text{eff}} = 0.1163 \hat{z} + 5.81 \times 10^{-3} \hat{x} \text{ mT} = 0.116 \text{ mT} \) at 2.9° tilt off the \( z \)-axis.

\[ \theta = \gamma B_{\text{eff}} T = 31.3 \text{rad} = 5 \text{ rotations around } B_{\text{eff}} \text{ NOT A } 90° \text{ pulse!} \]

2-a) \( \gamma_c = 2.8 \times 10^{10} \text{ Hz/T} \). At 1.5 T, \( \nu = 42.0 \text{ GHz} \). Is this practical? On chemical samples yes – ESR is a common technique. However, this frequency is in the microwave range, so this is not practical for humans \( \Rightarrow \) will potentially have a great deal of energy deposition in your subject (this is BAD!)

2-b) In 1.5 T NMR experiment, our Larmor frequency is 64.5 MHz. To generate 64.5 MHz in an ESR experiment, we need a 2.3 mT field \( (B = \frac{\nu}{\gamma}) \).

2-c) Using our expression from 1-a, we can calculate that we would a \( B_1 = 8.93 \times 10^{-7} T \) to generate the desired RF pulse.
3-a) \( \frac{n_\downarrow}{n_\uparrow} = \exp\left(-\frac{\gamma B h}{K_B T}\right) \) is the expression for the difference in spin population levels based on a Boltzmann distribution (again, watch your units for \( \gamma \) and \( h \) – if you use one in angular form, they both must in angular form).

<table>
<thead>
<tr>
<th>Field Strength (T)</th>
<th>Ratio for (^1)H</th>
<th>Ratio for (^{13})C</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.0</td>
<td>0.99995</td>
<td>0.99999</td>
</tr>
<tr>
<td>3.0</td>
<td>0.99998</td>
<td>0.999995</td>
</tr>
<tr>
<td>1.5</td>
<td>0.99999</td>
<td>0.999997</td>
</tr>
</tbody>
</table>

So net magnetization increases with field strength, but we are still dealing with incredibly small signals!

3-b) If we re-arrange our initial expression, to get 1:2 ratio of spins, we need temperatures of 20.6 mK for \(^1\)H and 5.2 mK for \(^{13}\)C.

3-c) For \(^1\)H, we’d need a field of 9.84 x 10\(^{-4}\) T to attain a 1:2 ratio of spins. For \(^{13}\)C, we’d need a field of 3.95 x 10\(^{-5}\) T. Clearly, these are not attainable fields in a laboratory setting.