1. Harmonic series.

A. What are the first 10 harmonics of a 200 Hz fundamental (F0)?

1st harmonic (fundamental): 200 Hz, 2nd: 400 Hz, 3: 600 Hz, 4: 800 Hz, 5: 1000 Hz, 6: 1200 Hz, 7: 1400 Hz, 8: 1600 Hz, 9: 1800 Hz, 10: 2000 Hz.

B. Which partials can be "heard out" or "resolved" as individual pitches?

(Partials are not divisible by whole numbers, usually interpreted by auditory system as noise). The partials that are harmonics of lower frequencies can be “resolved” as individual pitches. For example, the 1.5 partial of 200 Hz is the same as 3rd harmonic of 100 Hz (300 Hz in both cases), so it can be resolved.

C. What would be the first 10 subharmonics of 200 Hz?

Subharmonic-frequencies below the fundamental, 1/(harmonic #).

1: 200 Hz, 2: 100 Hz, 3: 66.7 Hz, 4: 50 Hz, 5: 40 Hz, 6: 33.3 Hz, 7: 28.6 Hz, 8: 25 Hz, 9: 22.2 Hz, 10: 20 Hz

D. What would be the fundamental of 400, 500, 600 and 700 Hz?

100 Hz. They are the 4th, 5th, 6th, and 7th harmonics of the fundamental.

E. What are subharmonics 1-10 of 400, 500, 600, and 700 Hz.

*400 Hz- 1: 400 Hz, 2: 200 Hz, 3: 133.3 Hz, 4: 100 Hz, 5: 80 Hz, 6: 66.7 Hz, 7: 57.1 Hz, 8: 50 Hz, 9: 44.4 Hz, 10: 40 Hz

*500 Hz- 1: 500 Hz, 2: 250 Hz, 3: 166.7 Hz, 4: 125 Hz, 5: 100 Hz, 6: 83.3 Hz, 7: 71.4 Hz, 8: 62.5 Hz, 9: 55.6 Hz, 10: 50 Hz

*600 Hz- 1: 600 Hz, 2: 300 Hz, 3: 200 Hz, 4: 150 Hz, 5: 120 Hz, 6:100 Hz, 7: 85.7 Hz, 8: 75 Hz, 9: 66.7 Hz, 10: 60 Hz

*700 Hz- 1: 700 Hz, 2: 350 Hz, 3: 233.3 Hz, 4: 175 Hz, 5: 140 Hz, 6: 116.7 Hz, 7:100 Hz, 8:87.5 Hz, 9: 77.8 Hz, 10: 70 Hz

F. Which subharmonic(s) do the partials have in common?

G. How is this related to the fundamental frequency?
2. Common harmonics

We will assume that each note consists of a harmonic complex of harmonics 1-12.
Let the lower note F₀ = 200 Hz.
A musical interval of an octave in just tuning is a frequency ratio of 2/1.
A musical interval of a fifth in just tuning is a frequency ratio of 3/2.
A musical interval of a fourth in just tuning is a frequency ratio of 4/3.
A musical interval of a major third in just tuning is a frequency ratio of 5/4.

For 200 Hz:
Harmonics-1: 200 Hz, 2nd: 400 Hz, 3:600 Hz, 4:800 Hz, 5:1000 Hz, 6:1200 Hz, 7: 1400 Hz, 8:1600 Hz, 9:1800 Hz, 10:2000 Hz, 11: 2200, 12:2400
Subharmonics-1: 200 Hz, 2:100 Hz, 3: 66.7 Hz, 4: 50 Hz, 5: 40 Hz, 6: 33.3 Hz, 7: 28.6 Hz, 8: 25 Hz, 9: 22.2 Hz, 10: 20 Hz, 11: 18.2 Hz, 12: 16.7 Hz

A. What is the fundamental frequency of a note an octave above 200 Hz?
   \[ \frac{2}{1} \times 200 \text{ Hz} = 400 \text{ Hz} \] (2nd harmonic)
   How many harmonics do the two notes share? (harmonics 1-12)
   6 (400 Hz, 800 Hz, 1200 Hz, 1600 Hz, 2000 Hz, 2400 Hz)
   How many subharmonics do the two notes share above 30 Hz? What are they?
   4 (200 Hz, 100 Hz, 66.7 Hz, 33.3 Hz)

B. What is the fundamental frequency of a note a fifth above 200 Hz?
   \[ \frac{3}{2} \times 200 \text{ Hz} = 300 \text{ Hz} \] (3rd harmonic)
   How many harmonics do the two notes share?
   For 300 Hz-1: 300 Hz, 2: 600 Hz, 3: 900 Hz, 4:1200 Hz, 5:1500 Hz, 6:1800 Hz, 7:2100 Hz, 8: 2400 Hz, 9: 2700 Hz, 10: 3000 Hz, 11: 3300 Hz, 12: 3600 Hz
   4 (600 Hz, 1200 Hz, 1800 Hz, 2400 Hz)
   How many subharmonics do the two notes share above 30 Hz? What are they?
   3 (100 Hz, 50 Hz, 33.3 Hz)

C. What is the fundamental frequency of a note a fourth above 200 Hz?
   \[ \frac{4}{3} \times 200 \text{ Hz} = 266.7 \text{ Hz} \]
   How many harmonics do the two notes share?
   3 (800 Hz, 1600 Hz, 2400 Hz)
   How many subharmonics do the two notes share above 30 Hz? What are they?
   2 (66.7 Hz, 33.3 Hz)

D. What is the fundamental frequency of a note a major third above 200 Hz?
   \[ \frac{5}{4} \times 200 \text{ Hz} = 250 \text{ Hz} \]
   How many harmonics do the two notes share?
   2 (1000 Hz, 2000 Hz)
   How many subharmonics do the two notes share above 30 Hz? What are they?
   1 (50 Hz)

E. Do you see any patterns emerging here?
Yes, the number of shared harmonics is the total number of harmonics in a note (12) divided by
the number in the numerator of the frequency ratio. For example, the shared harmonics for a note
a fifth above is \(12/(\text{numerator in frequency ratio})=12/3=4\). Likewise, the number of subharmonics shared is \(5-(\text{number in the denominator of the frequency ratio})\) as 5 may symbolize halfway point of the octave. total number thirds (the major unit, as it is the smallest harmonic in this example and the harmonic with the lowest number that is consonant, unlike the 2nd)?

### 3. Tuning systems

We have decided to devise a tuning system in which we divide the octave into 12 logarithmically spaced semitones (equally temperament tuning, chromatic scale). There are several ways of doing this. One is to take the twelfth root of 2. The first note in the scale is 1, the second note is \(2^{1/12}\) the third \(2^{2/12}\) the fourth \(2^{3/12}\) and so on up to the octave \(2^{12/12}=2\). These frequency ratios are multiplied by the frequency of the first note in the scale (the tonic). Frequency ratios can be expressed in cents. There are 1200 cents to the octave. A semitone is 100 cents, a perfect fifth is 800 cents, a tritone is 600 cents.

A. If (F0) note A = 440 Hz, what are the frequencies of the other notes in the octave that spans 440 to 880 Hz? What are the frequencies in the octave below it?

B. How big is a semitone (one step in the scale) in terms of percent change in frequency? What percent change in frequency is 50 cents? 25 cents?

### Equations:

\[
A = B \times 2^{n/1200} \text{ for finer calculations}
\]

\[
N = \log_2(A/B) \text{ approximately equal to } N = \log_{10}(A/B) \text{ where } N \text{ is the number of cents}
\]

A semitone is a half step on the scale (100 cents), so the percent change in frequency is \(A/B = 2^{1/12} = 1.047\). 1.047-1=.047 or 4.7%.

Alternatively, 100/1200=1/12=.083 or 8.3%.

C. In percentage terms, how much does the equally tempered scale differ from just intonation for the fifth (3/2), fourth (4/3)? You can calculate this out or consult the table in Handel, p. 331. If the jnd for fundamental frequency is in the neighborhood of 0.2% under the best of conditions, do you expect this difference to be audible? Where is the greatest discrepancy between intervals based on just intonation and those based on equal temperament?

For the fifth, the equally tempered scale has a frequency ratio of 1.498 and corresponds to 700 cents, versus 1.500 and 702 for the just intonation scale. The equally tempered scale has a lower frequency ratio by .002 and is 2 cents less. No, it is not likely audible as the difference is less than a tenth of the just noticeable difference under the best of conditions. The greatest discrepancy between the two scales occur at the dissonant intervals: major second D and minor seventh A for around 20 cents difference or tritone F# and minor seventh A# with a frequency ratio difference of 0.018 each.

D. If one were to devise a tuning system that divided the octave into 10 logarithmically equal steps, what would the frequencies be for the octave that spans 440-880 Hz? How far would these notes diverge from those of just-intonation? William Sethares in his book *Tuning, Timbre, and Spectrum* discusses such a scale and has written musical compositions based on it. The frequencies would be 44 Hz apart so 484, 528…

### 4. Major aspects of music
A. Which aspects of music depend (most directly) on perception of pitch? (Two or more examples)
Frequency and period is dependent on the perception of pitch and are hard to measure otherwise. Sound pressure of the tone and spectrum or harmonic spacing are other aspects. Rate and place of stimulation is affected by pitch, as seen in some cochlear implant studies.

B. Which aspects of music depend on perception of distinct events?
Echoes serve as an aspect that depends on perception of distinct events. Music depends on perception of distinct events when there are dissonant occurrences, like two sources that produce different sounds (vocals and bells) or a violinist who does not sync up to the rest of the orchestra.

C. Which aspects of music depend on memory processes (musical contexts, kinds of expectations)?
Expectations of timing and rhythm may depend on memory processes. Music associated with traditions (such as the wedding march or military fanfares) or widely used musical pieces (like the religious “Amazing Grace”) can depend on context. For example, the wedding song and “Amazing Grace” are thought to be slower and more calming than the more bright brass notes of the military wake-up trumpet call).

5. Pitch and the limits of musical tonality
A. Roughly, what is the range of pure tone frequencies that we can hear?
We can hear the frequencies from about 20 to 20,000 Hz.

B. What is the range of pitch (f, F0) frequencies for musical tonality (i.e. over which octaves, musical intervals, and melodies can be readily recognized).
Most of music occurs within two octaves in the frequency range of 220 to 880 Hz. Components of an orchestra (brass, winds, and strings) produce sounds within this range. The range of vocal perception a bit larger in the 30 to 5000 Hz.

C. What would constitute an explanation (neural or psychological) for why these ranges are different?
The range of frequencies we can hear and those that are observed in easily recognized melodies are different because we are only able to make meaning of certain patterned sounds, such as those that contain uniform waveforms and expected, ‘musically pleasing’ note and rhythm combinations. We may hear static or alarm noises at lower or higher frequencies but cannot construct a meaningful interpretation.

D. Provide a possible (neural or psychological) explanation for why we hear the missing fundamental.
The phenomenon that Licklider described for hearing the missing fundamental may be the ability of the higher cognitive functions to perceive pitch by subtracting frequencies of the harmonic (instead of hearing each harmonic separately), resulting in filling in of the missing fundamental (a part of the expected harmonic pattern). Neurologically, some theories point to the ability to infer activity from the patterns of the basilar membrane.
E. What does it mean to say that musical pitch is highly invariant with respect to tone intensity, location, and duration? What is metamery? What would be an example of pitch metamery?
Musical pitch is not dependent to tone intensity, location, or duration as it is dependent on frequency; a C sharp sounds like a C sharp no matter how intensely it is played on the piano, if it is played an octave higher, or if it is held onto longer. An equation confirms this: I=P/A (amplitude does not affect frequency, only wavelength and speed of sound).

6. Timbres of musical instruments
A. What is the main functional role that timbre plays in most western tonal music?
Timbre gives music texture in tonal music. It is similar to the onset of homophony and polyphony after the monophony of chants in Western music, which gave more rich texture with more vocal or instrumental parts.

B. What is the acoustic dimension that distinguishes timbres of stationary sounds?
Stationary sounds do not refer to temporal qualities (timing, such as the envelope qualities of attack time and decay) that distinguish between the different sound textures made by instruments.

C. What are two factors that distinguish timbres of non-stationary (time-varying) sounds?
Spectrum and envelope (coloration, tonal character, vibratos, tremolos, etc) are used to perceive timbre.

Classes of instruments: 1. pipe organ (vibrating air column), 2. violin (bowed string), 3. piano (struck string), 4. bells (struck, metal), 5. snare drum (struck, membrane)

D. Which classes produce harmonic tones? Inharmonic tones?
Harmonic tones are produced by violins, pianos, and pipe organs. Inharmonic tones can be created by bells and snare drums.

E. Which classes can produce a melody? Rhythm?
Pipe organs, violins and pianos can produce a melody. Bells and snare drums can produce rhythms. Bells may also be able to produce rhythm if there is a series of bells with different pitches (similar to a xylophone).

7. Scales and musical intervals
A. What is a scale?
A scale is a series of musical notes in ascending or descending order, usually with the same step size between each note.

B. What role do scales play in music perception?
Scales give a dimension of order in music perception and serve as the basis for how we perceive pitch. For example intervals and perception in differences between harmonics can be better understood if one has heard scales played, especially on a piano.

C. What is the tonic? What functional role does the tonic play in perception of melody and harmony?
A tonic is the first note in a musical scale. The musical piece is based on the tonic, and the chord including the tonic is also usually prevalent; it can be thought of as the 'home key.'
harmony of the piece may change to different keys and produce different moods, and the tonic serves a benchmark for comparison. For example, a symphony may start in C minor and switch to E flat major during the exposition or development and conclude in the tonic key of C minor.

D. Why do melodies played in different modes have distinctive qualities, despite their use of the same set of notes?
Melodies in different modes have distinctive qualities because the pitch relationships between the notes may be somewhat different. Comparing major and minor modes, F in the major mode includes an F# while that of F minor includes the white key. The medieval modes also exemplify this; the fourth interval is augmented in Lydian mode while it is perfect in the other modes, the Locrian fifth is diminished while the others are perfect, and there is a mix of major and minor intervals (except for the prime and octave) in the different modes.

8. Consonance
A. What defines a musical interval? Does an interval depend on the absolute (F0) frequency of its root?
A musical interval is defined by the space (number of half steps) between one note and the other note or notes. An interval does not depend on the absolute frequency of the root.

B. "Consonance" is associated with 1) euphony and tonal preference on one hand and 2) stability/instability and tension/resolution on the other. Dependence of consonance on musical context notwithstanding, which isolated intervals (octaves, seconds, fourths, fifths, sevenths) are perceived as more consonant or dissonant?
Octaves, thirds, fourths, and fifths are interpreted as more consonant, while seconds and sevenths are interpreted as more dissonant.

C. Two rival theories of consonance were proposed by Helmholtz and Stumpf in the 19th century. What perceptual aspect of tone combinations does each rely on?
Helmholtz proposed that the more mismatched the harmonic (the more beats between the notes), the more dissonant the sound, which explains why whole whole tones<perfect fifths<octaves in consonance. On the other hand, Stumpf based his theory of consonance and dissonance on the fusion and nonfusion, respectively, of tonal combinations. Helmholtz’s theory depends on beats while Stumpf’s relies on fusion.

D. Which aspect is thought to be the result of the beating of nearby harmonics? of waveform regularity and its consequences for pitch?
Dissonance is thought to be the result of beating of harmonics nearby. Waveform regularity and more uniform distribution of pitch contribute to consonance.

9. Harmony and harmonics
A. What notes and musical intervals constitute a major triad chord (I) in the key of C (major)?
What notes and musical intervals constitute a minor triad chord (I) in the key of C (minor)?
For a major chord in C major: C, E, and G. C and E, as well as E and G, correspond to a third. C and G correspond with the interval of a fifth.
For C minor: C, E flat, and G. The intervals are the same as those for C major.
B. Tonal hierarchy of notes. In the context of Krumhansl's probe-tone studies, we discussed tonal hierarchies based on 1) the tonic, 2) notes in I - triad, 3) other notes in the diatonic scale, and 4) other notes in the chromatic scale. Which notes would fall into these respective classes for the key of C major?

1) C serves as the tonic note. 2) C, E, and G are in triad I. 3) C D E F G A B are in the diatonic scale and 4) C  C# D  D#  E  F  F#  G  G#  A  B flat  B.

For simplicity assume that the tonic (F0) of a just-tuned diatonic scale is 200 Hz and the notes of the scale consist of harmonics 1-6 (consult Handel, p. 331 for just temperament F0 ratios, p. 364 for chord notes).

C. What would be the fundamental frequencies (F0s) of the notes in the major triad (I) if the root is 200 Hz?
If 200 Hz is the frequency of note C, E is 5/4*200=250 Hz and G is 3/2*200=300 Hz.

D. What harmonics would be present? How many pairs of common harmonics (overlaps) are there?
The third and the fifth harmonics would be present. There is a pair of third harmonics (the C to E and E to G).
If question is asking about the scale, all 6 harmonics would be present and there are 5 2\textsuperscript{nd} harmonics, 4 3rds, 3 4ths, 2 5ths and 1 6\textsuperscript{th}.

E. What would be the fundamentals of the notes in the minor triad (I)? How many harmonic overlaps would there be?
The fundamentals of the notes and the harmonic overlaps would be approximately the same, but having the E flat instead of the E corresponding to a white key would alter the calculations by a half step more for the C to E flat 3\textsuperscript{rd} harmonic and half a step less for the E flat to G third.

In major keys, chords of scale degrees I, IV, and V have major interval patterns (Major 3rd + Minor 3rd), while other scale degrees II, III, VI, VII have minor interval patterns (Minor 3rd + Major 3rd). If chord stability, consonance, and pitch unity were determined solely by the proportion of coincident harmonics, would you expect the major or minor triad pattern to be more stable? Why?
I would expect the major triad to be more stable because there is more symmetry in the chord patterns (no half step differences between the third harmonics). Also, I, IV, and V have more consonance than the minor scale degrees (especially the II and VII).

If chord stability were determined by the strength of the fundamental bass, which in turn were determined by the number of coincident subharmonics, which pattern would be more stable? In this case, perhaps the minor triad pattern would be more stable because it may have more coincident subharmonics and therefore a stronger fundamental base and greater stability (unless the shared subharmonics for I and V are greater than those shared between II, III, and VII).

10. Chord progressions
A. What is a chord progression?
Chord progression is a series of chords played according to the order of a scale.

B. In musical contexts, chord stability is also influenced by its relation to the tonic. If degree of relatedness between a chord (II-VII) and the tonic chord (I) of the key (Krumhansl et al, 1982, Handel, p. 365) determines the perceptual "stability" of that chord, then what pattern of movement would be expected for a I-IV-VII-I progression? Assume that movement from a less stable chord to a more stable one is "resolution" (R), while the opposite movement builds "tension" (T), e.g., a I-IV-II-V-I produces a pattern of movement of T-T-R-R, see Handel, p. 341).

\[ T-T-T-R \]

C. What is key modulation?
Modulation is the process of changing from one key to another, sometimes the same key an octave higher or lower (so key signature does not change though it can in modulation in general).

D. Why might perceived relatedness of chords depend on key context? (Handel, p. 373)
There is an element of expectation that the context key engenders; relevant notes and triads to the key context sound consonant while others sound "jarring". Spatial differences of chords within the contexts (versus no context) are also important.

11. Melody
A. What is a melody?
A melody is a linear combination of musical tones that is thought of as a unified entity.

B. What does it mean to say that melody is invariant under transposition?
The melody stays the same no matter what key it is played in.

C. What distinguishes a tonal from an atonal melody (see Handel, pp. 355-361)?
Tonal melodies are perceived to have a single theme because of sequential contours of a melody (the amplitude up and down or +/- patterns), size and direction of semitone intervals between successive notes, the pattern of scale steps in a tonal framework, and the actual notes played in the melody.

D. Which is more important for melody recognition in tonal contexts: absolute pitch, pitch contour sequences of musical intervals, relations of notes to the tonic?
Relation of the notes to the tonic is more important for melody recognition in tonal contexts.

E. What are the shortest notes that can support a good melody (ballpark)? the longest notes?
The shortest notes are 900 Hz and the longest 200 Hz.

12. Rhythm
A. Define tempo, accent, beat, meter, and rhythm.
Tempo-speed or pace of a composition
Accent-an emphasis placed on a certain musical note, played louder and with faster onset
Beat-a basic time unit used in music
Meter- a length measurement of stressed and unstressed beats
Rhythm-timing of a series of notes
B. How does rhythm differ from melody? How are these two aspects of music similar? How are they different?
Rhythm focuses on temporal components while melody focuses on pitch relations, though there is still an element of rhythm in melody.

C. Why is accent important for perception of rhythmic pattern?
Accents call attention to certain beats and sometimes changes in rhythm, which may help guide a listener rhythmically. For example, in the beginning of Stravinsky’s “Rite of Spring” gives an accent every six beats to give a sense of progression of tenseness and conflict.

D. If tempo is set at 1 quarter note = 1 beat at 120 bpm, how long in absolute terms is a quarter note? a 16th note?
A half note (?) would be 120*2=240 bmp and a 16th note is 120*2*2 (quarter→eighth→sixteenth)=480 Hz.

E. How do the note durations needed to support a good melody relate to those needed for a good rhythm?
Note durations can add to the sentiments or mood of a piece. For example, a fermata may give a sense of closure to a piece or movement of a piece.

13. Organization of voices (auditory scene analysis)
A. What acoustical factors influence whether two instruments will be perceptually fused (heard as one voice)?
The pitch or frequency range and timbre, as well as rhythmic synchronicity and beat patterns, influence whether instruments are thought of as fused.

B. What distinguishes a homophonic from a polyphonic piece of music? What musical factors (melody, harmony, rhythm) influence whether a musical piece is (perceived as) one or the other?
A homophonic voice is centered on one melody and rhythm, even if there are several voices contributing to the harmony (ie an acapella group). A polyphonic piece has greater texture because the different parts or voices may enter at different times with varying rhythms and harmonies.

14. Innate vs. learned mechanisms. In your opinion, which aspects of music perception and cognition are likely to depend on bottom-up mechanisms of auditory perception? Which aspects are likely to depend on prior auditory experience that is not culture specific? Which are likely to depend on culture-specific auditory experiences, i.e. cultural conditioning? (Short paragraph, briefly give reasoning.)
Bottom-up mechanisms that just rely on sensory inputs may include timbre, as it seems to be recognized very easily and there are no circumstances I know of that can impair somebody’s ability to tell apart the sounds of a waterfall or drum. Rhythm may also be in this category but there are instances where rhythm may appeal to a higher process of expectation. For example, Parkinson’s disease or stroke patients can improve their gait or be roused from their catatonia, in the case of PD. Autistic individuals often respond to the rhythmic components in music therapy. Those that are deaf (and thus without the bottom up processes involved in hearing) are likely to
recognize patterns in metronome ticking or ebbing and flowing of the tide. I believe having relative ‘perfect’ pitch can be conditioned. The notion of consonance is likely inherent as it is immediately and universally recognized. Some cultures may have more dissonant sounding elements, most basic chants still have musicality.