MIT 3.071
Amorphous Materials
8: Mechanical Properties

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After-class reading list

- Fundamentals of Inorganic Glasses
  - Ch. 18
- Introduction to Glass Science and Technology
  - Ch. 9
**Glass = fragile?**

<table>
<thead>
<tr>
<th>Material</th>
<th>Iron</th>
<th>Structural steel</th>
<th>Glass fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate tensile strength</td>
<td>35 MPa</td>
<td>550 MPa</td>
<td>4890 MPa</td>
</tr>
</tbody>
</table>

- Iron man
- Glass
Strength and toughness

- Strength: applied stress a material can withstand
- Toughness: energy absorbed by (work performed to) a material per unit volume before fracture
Theoretical strength of a brittle material

- Theoretical strength is determined by the cohesive force between atoms.
- Work $W$ performed to separate the solid equals to the energy of the fresh surfaces created during fracture.

\[ \sigma = E \varepsilon \]
Theoretical strength of a brittle material

- Theoretical strength is determined by the cohesive force between atoms
- Work $W$ performed to separate the solid equals to the energy of the fresh surfaces created during fracture

$$\sigma = \sigma_m \sin \frac{\pi \varepsilon}{\lambda}$$

When $\sigma \ll \sigma_m$, $\sigma = E \varepsilon$

$$\sigma_m = \frac{\lambda E}{\pi}$$

Work $W$ performed:

$$W = V \cdot \int_0^\lambda \sigma d\varepsilon = \frac{2 \lambda^2 E V}{\pi^2} = 2 \gamma S$$

$$V = a_0 S \Rightarrow \lambda = \pi \sqrt{\gamma / E a_0}$$

$$\Rightarrow \sigma_m = \sqrt{\frac{\gamma E}{a_0}}$$
Theoretical strength of a brittle material

Consider silica glass

- $\gamma = 3.5 \text{ J/m}^2$, $E = 70 \text{ GPa}$, $a_0 = 0.2 \text{ nm}$

$$\sigma_m = \sqrt{\frac{\gamma E}{a_0}} = 35,000 \text{ MPa}$$

<table>
<thead>
<tr>
<th>Material</th>
<th>Glass</th>
<th>Silica glass</th>
<th>Silica nanowire</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate tensile strength</td>
<td>~ 30 MPa</td>
<td>110 MPa</td>
<td>26000 MPa†</td>
</tr>
</tbody>
</table>

Practical strength of engineering materials is much less than their theoretical strength

Griffith’s theory

- Strength of practical materials is limited by stress concentration around tiny flaws (Griffith cracks)
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Stress concentration factor:

\[ \frac{\sigma_{\text{max}}}{\sigma_\infty} = 2\sqrt{\frac{a}{\rho}} = 2\sqrt{\frac{a}{a_0}} \]

Fracture strength of a flawed material:

\[ \sigma_f = \frac{1}{2} \sqrt{\frac{\gamma E}{a}} \]
Griffith’s theory

- Strength of practical materials is limited by stress concentration around tiny flaws (Griffith cracks)

In flawed silica glass:

\[ \sigma_f = \frac{1}{2} \sqrt{\frac{\gamma E}{a}} = 110 \text{ MPa} \]

\[ \Rightarrow a = 5 \text{ } \mu\text{m} \]

Visualizing Griffith cracks in glass

Displacement field near a crack tip

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Stress intensity factor and fracture toughness

- Stress intensity factor (tensile):
  \[ K_I = \sigma_\infty \sqrt{\pi a} \quad K_{Ic} = \sigma_f \sqrt{\pi a} \]
  critical stress intensity factor

- Strain energy release rate:
  \[ G_I = \frac{K_I^2}{E} \quad G_{Ic} = \frac{K_{Ic}^2}{E} \]
  work of fracture

- Fracture condition:
  \[ K_I > K_{Ic} \quad G_I > G_{Ic} \]

\[ K_{Ic} \] is a material constant and is independent of crack length
Intrinsic plasticity in amorphous metals

- Lack of global plasticity
- Intrinsic plasticity
- When $G/K < 0.42$: plastic; $G/K > 0.42$: brittle ($K$: bulk modulus)

Brittle fracture of glass

- When a crack exceeds the critical length, the crack becomes unstable and propagates catastrophically through the material.

Crack propagation velocity:

1540 m/s


Glass cracking at 231,000 fps
Fractography

Conchoidal fracture

Image from "Fracture analysis, a basic tool to solve breakage issues"
Static fatigue in glass

- Under constant load, the time-to-failure varies inversely with the load applied.

Sub-critical crack growth: crack length increases over time even when $\sigma_\infty < \sigma_f$. 
Stress corrosion

- Reaction at crack tip:
  \[-\text{Si-O-Si}^- + \text{H}_2\text{O} \rightarrow \text{Si-OH} + \text{HO-Si}^-\]

- Higher alkaline content generally reduces fatigue resistance

- Higher susceptibility to stress corrosion in basic solutions

- Thermally activated process


Fracture toughness measurement

- Standard specimen geometries to obtain load-displacement plot

![Compact tension specimen](image1.png)

![Single edge-notched bend specimen](image2.png)

![Middle-cracked tension specimen](image3.png)

Indentation of glass samples

- Mechanical properties evaluated through indented crack size or crack-opening displacement based on empirical equations
- Poor correlation with conventional test results can be a concern

Indentation of glass samples

- Vickers indentation of soda-lime glass

Fracture statistics

- Experimental results of fracture strength can often be described by the Weibull distribution.
- The fraction $F$ of samples which fracture at stresses below $\sigma$ is given by:
  \[
  F = 1 - \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]
  \]
  $m$: Weibull modulus
- Probability density $dF/d\sigma$
  - Probability of samples fracture at stress $\sigma$
Weibull plot

\[ F = 1 - \exp \left[ -\left( \frac{\sigma}{\sigma_0} \right)^m \right] \]
\[ \Rightarrow \ln \left[ -\ln (1 - F) \right] = m \left( \ln \sigma - \ln \sigma_0 \right) \]

\[ m \text{ : slope of the Weibull plot} \]
\[ \sigma_0 \text{ : intercept with horizontal axis} \]

Summary

- Theoretical and practical strengths of materials
  - Practical strength of brittle materials is usually much lower than the theoretical strength due to the presence of defects
  - Oxide glasses are extremely sensitive to surface defects
  - Intrinsic ductility in select BMGs contributes to high toughness

- Basics of fracture mechanics
  - Griffith crack theory
  - Fracture toughness \( K_{lc} = \sigma_f \sqrt{\pi a} = \frac{1}{2} \sqrt{\pi \gamma E} \)

- Fatigue and stress corrosion
- Fracture toughness measurement
- Fracture statistics: Weibull plot