Problem #1:

- Symmetries ✓
- Labels ✓
- Planes ✓
Problem #2:

\[
\begin{pmatrix}
1 \\
1 \\
\end{pmatrix} \times \begin{pmatrix}
\bar{1} \\
0 \\
\end{pmatrix} = \begin{pmatrix}
1 \\
\bar{2} \\
1
\end{pmatrix}
\]

- Cross prod.
- Results for \( \Pi \)
- Directions in proj.
Problem #3:

- 4 grains ✓
Problem #4:

\[ \gamma = \frac{\mu b}{2\pi r} \quad \Gamma = \frac{\mu b}{2\pi r} \quad \gamma_{\text{crit}} = 2 \text{ MPa} \]

<table>
<thead>
<tr>
<th></th>
<th>AL</th>
<th>Au</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu/\text{GPa})</td>
<td>25</td>
<td>27</td>
</tr>
<tr>
<td>(b/\text{nm})</td>
<td>0.283</td>
<td>0.288</td>
</tr>
</tbody>
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\[ \Rightarrow \text{AL: } \gamma_{\text{crit}} = \frac{25 \cdot 10^{8} \text{ N/m}^2 \cdot 0.283 \cdot 10^{-9} \text{ m}}{2 \pi \cdot 2 \cdot 10^{6} \text{ N/m}^2} = 0.56 \mu m = 1.98 \cdot 10^{3} \mu m \]

\[ \Rightarrow \text{Au: } \gamma_{\text{crit}} = \frac{27 \cdot 10^{8} \text{ N/m}^2 \cdot 0.288 \cdot 10^{-9} \text{ m}}{2 \pi \cdot 2 \cdot 10^{6} \text{ N/m}^2} = 0.62 \mu m = 2.15 \cdot 10^{3} \mu m \]

\[ \gamma_{\text{crit}, \text{Au}} > \gamma_{\text{crit}, \text{AL}} \]

- equation \(\checkmark\)
- find properties \(\checkmark\)
- calculate \(\gamma_{\text{crit}}\) and conclusion \(\checkmark\)
Problem #5:

a) No change in distance since stress field is the same
- Parallel screws => repel => 2nd screw moves in same direction
- Antiparallel attract => 2nd screw moves in opposite direction => annihilation

\[ \tau_{xy} = \frac{\mu b x}{2\pi (1-\nu)} \frac{(x^2 - y^2)}{(x^2 + y^2)^2} \quad y = 0 \text{ same plane} \]

\[ \tau_{xy} = \frac{\mu b x}{2\pi (1-\nu)} x = \frac{\mu b}{2\pi (1-\nu) x} \Rightarrow x = \frac{\mu b}{2\pi (1-\nu) \tau_{xy}} \]

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<tr>
<td>(r)</td>
<td>0.35</td>
<td>0.42</td>
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\[ \Rightarrow \text{Al:} \quad x = \frac{25 \cdot 10^9 \text{N/m}^2 \cdot 0.283 \cdot 10^{-9} \text{m}}{2\pi (1-0.35) \cdot 2 \cdot 10^{6} \text{N/m}^2} = 6.1 \cdot 10^{-3} \text{b} \]

\[ \Rightarrow 1.7 \mu \text{m} \]

\[ \Rightarrow \text{Au:} \quad x = \frac{27 \cdot 10^9 \text{N/m}^2 \cdot 0.288 \cdot 10^{-9} \text{m}}{2\pi (1-0.42) \cdot 2 \cdot 10^{6} \text{N/m}^2} = 6.6 \cdot 10^{-3} \text{b} \]

\[ \Rightarrow 1.9 \mu \text{m} \]

\( x \approx 3 - 4 r_{\text{crit}} \Rightarrow \text{edge bigger stress field than screw} \)

c) Edge and screw have complimentary stress fields => edge not moved by screw

- Discussion of effects in a) ✓
- Comparison screw edge ✓
- Complimentary stress fields ✓
Problem #6:

- Decompose $\vec{b}$ into screw and edge as a function of the rotation angle of the loop
  
  $$\vec{b} = \vec{b}_{\text{screw}} + \vec{b}_{\text{edge}}$$

  \[ b_{\parallel} = |\vec{b}| \cos \theta \quad \Rightarrow \quad b_{\parallel} = |\vec{b}| \cos \theta \]

  $$b_{\perp} = |\vec{b}| \sin \theta \quad \Rightarrow \quad b_{\perp} = |\vec{b}| \sin \theta$$

- $dE_{\text{screw}} = \mu |\vec{b}_{\text{screw}}|^2 \, dl \Rightarrow \quad dE_{\text{screw}} = \mu (|\vec{b}| \cos \theta)^2 \, r \, d\theta$

  \[ dl = r \, d\theta \]

  \[
  \begin{align*}
  E_{\text{screw}} &= \int_0^{2\pi} \mu |\vec{b}|^2 \cos^2 \theta \, r \, d\theta \\
  &= \mu |\vec{b}|^2 r \int_0^{2\pi} \cos^2 \theta \, d\theta \\
  &= \mu |\vec{b}|^2 r \left[ \frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{2\pi} \\
  &= \mu |\vec{b}|^2 r \pi \\
  \end{align*}
  \]

- $E_{\text{edge}} = \mu |\vec{b}|^2 r \pi$

- $E_{\text{tot}} = E_{\text{edge}} + E_{\text{screw}} = \pi \mu |\vec{b}|^2 r \frac{2 - \frac{r}{2}}{1 - \frac{1}{2}}$

- Decomposition of $\vec{b}$
- Integral + Equations
- Total Energy of Screw
3.40J / 22.71J / 3.14 Physical Metallurgy
Fall 2009

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