Prob. 19.21 - Cantilevered beam

The stress \( \sigma \), has no dependency on the material properties, and is not influenced by material viscoelasticity.

The correspondence-principle recipe starts by putting the deflection relation in the Laplace plane (\( C \) is the compliance operator):

\[
\bar{v} := \frac{1}{6} \frac{x^2 (3L-x) CF_{\text{bar}}}{Ix}
\]

Transform of load:

\[
\text{Transform of load:}
\]

\[
\text{with(inttrans): F_{\text{bar}} := laplace(F*Heaviside(t),t,s);}
\]

\[
F_{\text{bar}} := \frac{F}{s}
\]

Compliance operator:

\[
\text{Compliance operator:}
\]

\[
C := C_g + \frac{C_v}{\tau \left( s + \frac{1}{\tau} \right)}
\]

Invert to get deflection in time plane:

\[
\text{Invert to get deflection in time plane:}
\]

\[
\text{v(t)} := \text{invlaplace(v_{bar}, s, t)};
\]

\[
v(t) := \frac{1}{6} \frac{x^2 (3L-x) F \left( -C_v \frac{t}{\tau} + C_g + C_v \right)}{Ix}
\]

Simplifying manually to standard form:

\[
v(x,t) = \frac{x^2 (3L-x)}{6I} \cdot F \cdot \left[ C_g + C_v \left( 1 - e^{-t/\tau} \right) \right]
\]

Superposition approach: write load and compliance as time functions:

\[
F := (t) \rightarrow F_0 \cdot \text{Heaviside}(t);
\]

\[
F := t \rightarrow F_0 \cdot \text{Heaviside}(t)
\]

\[
C_{\text{crp}} := (t) \rightarrow C_g + C_v \left( 1 - e^{-t/\tau} \right);
\]

\[
C_{\text{crp}} := t \rightarrow C_g + C_v \left( 1 - e^{-t/\tau} \right)
\]

Superposition integral:

\[
\text{Superposition integral:}
\]

\[
v(t) := (x^2 (3L-x)/(6Ix)) \cdot \text{int}(C_{\text{crp}}(t-xi) \cdot \text{diff}(F(xi),xi), xi = -\text{infinity..t});
\]
\[
v(t) := -\frac{1}{6} x^2 (3 - x) F_0 \text{Heaviside}(t) \left( -C_g - C_v + C_v e^{-\frac{t}{\tau}} \right)
\]

This can be reduced to the same form obtained previously.

Examine deflection function for arbitrary choice of parameters:

\[
\text{plot}\left(\text{subs}\left\{L=2, x=1, F_0=1, C_g=1, C_v=1, \tau=1, I_x=1\right\}, v(t)), t=-5..5\right)
\]

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**Prob. 19.22 - Rigid die**

Define Poisson (N) and tensile modulus (EE) operators in terms of dilatation (K) and shear (G) operators:

\[
N := \frac{3K - 2G}{6K + 2G}; \quad EE := \frac{9GK}{3K + G};
\]

SLS expressions for G and K:

\[
G := Gr + \frac{(G_g - Gr) s}{s + (1/\tau_G)};
\]

\[
K := Kr + \frac{(K_g - Kr) s}{s + (1/\tau_K)};
\]
\[ K := Kr + \frac{(Kg - Kr) s}{s + \frac{1}{\tau_K}} \]

Pick model parameters from Fig. 17. Relaxation times (\(\tau_K\) and \(\tau_G\)) are those times at which the relaxation has dropped \((1/e)\) of its total value.

\[
\text{Digits}:=20; \\
Gg:=8.8*10^8; \\
Gr:=2.4*10^5; \\
\text{tau}_G:=.001; \\
Kg:=6.2*10^9; \\
Kr:=1.7*10^9; \\
\text{tau}_K:=.0005; \\
\text{sigybar}:=\sigma_y/s; \\
\]

Transverse stress:

\[
\text{sigxbar}:=N\text{sigybar}/(1-N) ; \\
\sigma_y:=10000000 ; \\
\text{sigma}[y]:=10*10^6; \\
\]

Note that the transverse stress becomes equal to the vertical stress (i.e. the stress state becomes hydrostatic) as the relaxation completes and the material approaches a rubbery state.

Vertical strain:

\[
\text{epsybar}:=(1+N)*(1-2*N)*\text{sigybar}/(EE*(1-N)); \\
\epsilon_y:=\text{invlaplace}(\text{epsybar},s,t); \\
\loglogplot(\epsilon_y,t=10^(-5)..10,labels=[t,`\epsilon_y`],numpoints=5000); \\
\]
We might expect the $(1-2\nu)$ factor to drive the strain to zero as $\nu$ approaches 0.5, but the tensile modulus is also dropping substantially, and the strain is observed to rise. The material does not become fully rubbery, and maintains a finite compressibility.