Problem 1. (15 pts). Estimate the following to two decimal places (show work).

\[
\begin{align*}
\sin (\pi + \frac{1}{10}) & = \sin (\pi) + \sin (\frac{1}{10}) \\
& = 0 - \frac{1}{10} = -0.10
\end{align*}
\]

b. (7 pts). \( \sqrt{101} = \sqrt{100 + 1} = \sqrt{100(1 + \frac{1}{100})} = 10\sqrt{1 + \frac{1}{100}} \leq 10 \left( 1 + \frac{1}{50} \right) = 10 + \frac{1}{5} = 10.02 \)

Problem 2. (20 pts). Sketch the graph of \( y = x^2 + 1 \) on \(-\infty < x < \infty\) and label all critical points and inflection points with their coordinates on the graph along with the letter "C" or "I".

Problem 3. (20 pts). An architect plans to build a triangular window with a base of two sides and a wall on the third side. Such that the base segments have fixed length \( b \). What is the length \( a \) of the third side if the region enclosed has the largest possible area? Show work and include an argument to show that your answer really gives the maximum area.

\[
\begin{align*}
A &= \frac{1}{2} a b \sin \theta, \quad \text{and } a^2 + (\frac{b}{2})^2 = L^2, \quad 0 \leq x \leq 2L.
\end{align*}
\]

**Method 1** (Substitution)

\[
A = \frac{1}{2} \sqrt{b^2 - x^2}, \quad A = \frac{1}{2} \times \sqrt{b^2 - x^2}
\]

\[
A^2 = \frac{1}{4} \left( \frac{b^2 - x^2}{b^2 - x^2} \right) = \frac{1}{4} \left( \frac{b^2 - x^2}{b^2 - x^2} \right) = 0
\]

Hence \( L^2 = b^2 \), and \( \theta = \frac{\pi}{2} \).

Max because: \( A = 0 \) at both ends \( x = 0 \) and \( x = 2L \)

And \( A > 0 \) at the unique critical pt. \( \theta \) between.

So this critical pt. must be where max is achieved. (2nd deriv test is needed at \( \theta \).)

**Method 2** (Implicit Diff)

\[
2 a \sin \theta = 0 \Rightarrow a = \frac{b}{\sqrt{3}}
\]

\[
0 = A = \frac{1}{2} \times \left( b + \sqrt{b^2 - x^2} \right) = \frac{1}{2} \times \left( b + \sqrt{b^2 - x^2} \right) = \frac{b}{2} + \frac{x^2}{2} = \frac{x^2 + 2b}{2} \Rightarrow x = \sqrt{2} \sqrt{b}
\]

\( x = \sqrt{2} \sqrt{b} \Rightarrow \theta = \frac{\pi}{4} \Rightarrow x = \sqrt{b} + \frac{\sqrt{b}}{4} = \frac{\sqrt{b}}{4} \Rightarrow x = \sqrt{2} \sqrt{b} \Rightarrow x = \sqrt{2} \sqrt{b} \Rightarrow x = \sqrt{2} \sqrt{b} \)

(Reasoning for max is the same.)
Problem 4. (15 pts.) A rocket is launched straight up, and its altitude is \( h = 10t^2 \) feet after \( t \) seconds. You are on the ground 1000 feet from the launch site. The line of sight from you to the rocket makes an angle \( \theta \) with the horizontal. By how many radians per second is \( \theta \) changing ten seconds after the launch?

\[
\tan \theta = \frac{h}{1000} = \frac{10t^2}{1000} = \frac{1}{100}t^2
\]

\[
\frac{d\theta}{dt} = \frac{1}{\cos^2(\frac{\theta}{2})} \left. \frac{dt}{dt} \right|_{t=10} = \frac{1}{50} \cdot 10
\]

\[
\left. \frac{d\theta}{dt} \right|_{t=10} = \frac{1}{5} \cos^2 \frac{\theta}{2} = \frac{1}{2} \left( \frac{\theta}{2} \right)^2 = \frac{1}{10} \left( \frac{\pi}{2} \right)^2
\]

Problem 5. (20 pts.) Evaluate the following indefinite integrals.

a. \[
\int \sin(5x) \, dx = \frac{1}{5} \cos(5x) + C
\]

b. \[
\int x^2 \, dx = \frac{1}{3} x^3 + C
\]

Problem 6. (20 pts.) Suppose that \( f(x) = e^{\pi x} \), and \( f(0) = 10 \). One can conclude from the mean value theorem that

\[
\frac{f(b) - f(a)}{b-a} = f'(c) \quad \text{for some } c \in (a, b)
\]

for which numbers \( A \) and \( B \)?

\[
f'(x) = f(x) = 10e^{\pi x} > 0 \quad \text{for all } x
\]

\[
0 < c < 1 \implies e^c < e \quad \text{and} \quad 1 < e^c
\]

\[
f(c) = 10 + e^{\pi c} < 10 + e
\]

and

\[
f(c) = 10 + e^{\pi c} > 10 + 1 = 11
\]

\[
A = 11
\]

\[
B = 10 + e
\]