18.01 Single Variable Calculus
Fall 2006

18.01 Exam 4

Problem 1. (15 points) Evaluate $\int \frac{dx}{x(x+1)^2}$

Problem 2. (15 points) Evaluate $\int (\ln x) x^2 dx$

Problem 3. (20 points) Use a trigonometric substitution to evaluate $\int_0^1 \frac{dx}{(4+x^3)^{3/3}}$ (Be careful evaluating the limits)

Problem 4. a. (10 points) Find an integral formula for the arc length of the curve $y = 2\sqrt{x+1}$ for 0≤ x ≤ 1. Do not evaluate.

b. (10 points) Find an integral formula for the surface area of the curve in part (a) rotated around the *x*-axis. Simplify the integrand and evaluate the integral.

Problem 5. a. (7 points) Sketch the spiral r = θ^2_1 0 ≤ θ ≤ 3∏. Say how many times the curve meets the x-axis counting θ = 0 as the first times, and mark those points with X-s. (Your sketch need not be accurate to scale.)

b. (8 points) On your picture, shade in the region 0 ≤ r ≤ θ^2, 0 ≤ θ ≤ 2∏, and find its area.

Problem 6. a. (10 points) Find the equation in polar coordinates for the line y = x – 1 in the form r= $\int(\theta)$

b. (5 points) Find the range of θ for the portion of line y=x-1 in the range $0 \le x \le \infty.$ (It helps to draw a picture.)