Limits and Continuity - Week 2

Pset 4
Due October 8

(1) Page 138: 17, 18, 21 (1,1,and 2 points respectively)
(2) Let $A(x) = \int_{-2}^{x} f(t) \, dt$ where $f(t) = -1$ if $t < 0$ and $f(t) = 1$ if $t \geq 0$. Graph $y = A(x)$ for $x \in [-2,2]$. Using $\epsilon, \delta$, show that $\lim_{x \to 0} A(x)$ exists and find its value. (You may want to draw yourself a picture of $|A(x) - A(0)|$ by considering the appropriate regions on a t−y coordinate plane that contains the graph of $y = f(t)$. This will help you see geometrically how to write $\delta$ in terms of $\epsilon$.)
(3) Notes F.2:2
(4) Suppose that $g, h$ are two continuous functions on $[a,b]$. Suppose there exists $c \in (a,b)$ such that $g(c) = h(c)$. Define $f(x)$ such that $f(x) = g(x)$ for $x < c$ and $f(x) = h(x)$ for $x \geq c$. Prove that $f$ is continuous on $[a,b]$.
(5) Let $f(x) = \sin(1/x)$ for $x \in \mathbb{R}, x \neq 0$. Show that for any $a \in \mathbb{R}$, the function $g(x)$ defined by

$$g(x) = \begin{cases} f(x) & : x \neq 0 \\ a & : x = 0 \end{cases}$$

is not continuous at $x = 0$.
(6) page 145:5

Bonus: Let $f$ be a bounded function that is integrable on $[a,b]$. Prove that there exists $c \in \mathbb{R}$ with $a \leq c \leq b$ such that $\int_{a}^{b} f(x) \, dx = 2 \int_{a}^{c} f(x) \, dx$. 

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