THIRD MIDTERM
MATH 18.022, MIT, AUTUMN 10

You have 50 minutes. This test is closed book, closed notes, no calculators.

Name: __________________

Signature: __________________

Recitation Time: __________________

There are 5 problems, and the total number of points is 100. Show all your work. Please make your work as clear and easy to follow as possible.

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1. (20pts) For what values of $\lambda$ does the function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$,

$$f(x, y, z) = \lambda x^2 - \lambda xy + y^2 + \lambda z^2,$$

have a non-degenerate local minimum at $(0, 0, 0)$?
2. (20pts) Let \( f: \mathbb{R}^3 \longrightarrow \mathbb{R} \) be the function \( f(x, y, z) = x^2 - y^2 + z^2 \).

(i) Show that \( f \) has a global maximum on the ellipsoid \( 2x^2 + 3y^2 + z^2 = 6 \).

(ii) Find this maximum.
3. (20 pts)
(i) Switch the order of integration in the integral
\[ \int_0^3 \int_{x^2}^9 xe^{-y^2} \, dy \, dx. \]

(ii) Evaluate this integral.
4. (20pts) Let $W$ be the region inside the sphere $x^2 + y^2 + z^2 = 1$ and inside the cone $z^2 = x^2 + y^2$.
Set up an integral to calculate the integral of the function $yz$ over $W$ and calculate this integral.
5. (20 pts) Let $D$ be the region in the first quadrant bounded by the curves $x^2 - y^2 = 1$, $x^2 - y^2 = 4$, $xy = 1$ and $xy = 3$.
(i) Find $dudv$ in terms of $dxdy$, where $u = x^2 - y^2$ and $v = xy$.

(ii) Evaluate the integral
$$\iint_D (x^4 - y^4) \, dx \, dy.$$
18.022 Calculus of Several Variables
Fall 2010

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