1. Forward elimination changes $Ax = b$ to a row reduced $Rx = d$: the complete solution is

$$x = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$$

(a) (14 points) What is the 3 by 3 reduced row echelon matrix $R$ and what is $d$?

(b) (10 points) If the process of elimination subtracted 3 times row 1 from row 2 and then 5 times row 1 from row 3, what matrix connects $R$ and $d$ to the original $A$ and $b$? Use this matrix to find $A$ and $b$. 
2. Suppose $A$ is the matrix

\[
A = \begin{bmatrix}
0 & 1 & 2 & 2 \\
0 & 3 & 8 & 7 \\
0 & 0 & 4 & 2
\end{bmatrix}.
\]

(a) **(16 points)** Find all special solutions to $Ax = 0$ and describe in words the whole nullspace of $A$.

(b) **(10 points)** Describe the column space of this particular matrix $A$. “All combinations of the four columns” is not a sufficient answer.

(c) **(10 points)** What is the reduced row echelon form $R^* = \text{rref}(B)$ when $B$ is the 6 by 8 block matrix

\[
B = \begin{bmatrix}
A & A \\
A & A
\end{bmatrix}
\]

using the same $A$?
3. **(16 points) Circle the words** that correctly complete the following sentence:

(a) Suppose a 3 by 5 matrix $A$ has rank $r = 3$. Then the equation $Ax = b$

   ( always / sometimes but not always )

   has ( a unique solution / many solutions / no solution ).

(b) What is the column space of $A$? Describe the nullspace of $A$. 
4. Suppose that \( A \) is the matrix

\[
A = \begin{bmatrix}
2 & 1 \\
6 & 5 \\
2 & 4
\end{bmatrix}.
\]

(a) (10 points) Explain in words how knowing all solutions to \( Ax = b \) decides if a given vector \( b \) is in the column space of \( A \).

(b) (14 points) Is the vector \( b = \begin{bmatrix} 8 \\ 28 \\ 14 \end{bmatrix} \) in the column space of \( A \)?