18.075
Fall 2004

Number Systems
1. natural numbers: \( m = 0, 1, 2, \ldots \) (positive integers)
   - cannot be used to solve \( x + m = n \) where \( x \) unknown, \( m,n \) natural
2. integers: \( m = 0, 1, 2, \ldots \) \(-1, -2, \ldots\) (negative integers)
   - cannot solve \( mx = n \) \( x \) may not be an integer
3. rational numbers: \( \frac{m}{n} \); \( m,n \) integers
   - cannot solve \( x^2 = m \) \( m \) positive integer
4. real numbers: as points on a line
   - cannot solve \( x^2 + 1 = 0 \)
5. complex numbers: \( z = x + iy \) \( x,y \) real \( i : i^2 = -1 \)
   - points on a plane

Complex numbers can be thought of as vectors.

"Length" of vector: \( |z| = \sqrt{x^2 + y^2} \geq 0 \)

\[ z_1 = x_1 + iy_1 \]

Algebra of Complex Numbers
- addition: \( z_1 = x_1 + iy_1 \)
\[ z_2 = x_2 + iy_2 \]
\[ z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2) = x_1 + i \cdot 1 \cdot y_1 + x_2 + i \cdot 1 \cdot y_2 = (x_1 + x_2) + i(y_1 + y_2) \]
- multiplication:
\[ z_1 \cdot z_2 = (x_1 + iy_1) \cdot (x_2 + iy_2) = x_1x_2 - y_1y_2 + i(x_1y_2 + y_1x_2) \]
- division:
\[ \frac{z_1}{z_2} = \frac{z_1 \cdot \overline{z}_2}{|z_2|^2} = \frac{x_2 \cdot x_1 + y_1 \cdot y_2 + i \cdot x_1 \cdot y_2 - y_1 \cdot x_2}{x_2^2 + y_2^2} \]

Triangle inequality: \( |z_1 - z_2| \leq |z_1| + |z_2| \leq |z_1| + |z_2| \)
$z_1 = z_2 \iff x_1 = x_2, y_1 = y_2$, but only in Cartesian system

Polar Coordinates $(r, \theta)$

\[
x = r \cos \theta \\
y = r \sin \theta
\]

\[
z = r (\cos \theta + i \sin \theta), \quad r \geq 0, \quad r = |z|
\]

Specify range of $\theta$: $0 \leq \theta < 2\pi$ (possible)

Important: $\pi < \theta < 2\pi$