Case I

\[ \int_{-\infty}^{\infty} \frac{\text{Re}(z)}{\alpha(z)} \, dx, \quad M = N + 2 \quad (\text{so it converges}) \]

\[ \text{as} \quad \int_{-\infty}^{\infty} \frac{dx}{1 + xe} \]

Definition: \( f(z) \) tends to 0 uniformly for \( \theta_1 < \theta < \theta_2 \) as \( R \to \infty \)

\[ \int_{C_R} f(z) \, dz \to 0 \quad (R \to \infty) \]

where \( C_R \) is a circle of radius \( R \), \( |z| = R \), \( \theta \leq \theta \leq \theta_2 \)

Theorem 1: If \( \forall R \in \mathbb{R}^+ \), \( f(z) \to 0 \) uniformly as \( \arg(z) = R \to +\infty \), then

\[ \lim_{R \to \infty} \int_{C_R} f(z) \, dz = 0 \]

where \( C_R \) is a circular arc, \( |z| = R \), \( \theta_1 \leq \theta \leq \theta_2 \)

\[ |z| \to 0 \]

Case II

\[ \int_{-\infty}^{\infty} \frac{dx}{\alpha(x)} \quad \alpha: \text{real} \]

Theorem 2: If \( f(z) \to 0 \) uniformly as \( |z| \to \infty \), then

1. \( a > 0 \), \( \lim_{R \to \infty} \int_{C_R} e^{az} f(z) \, dz = 0 \)
2. \( a < 0 \), \( \lim_{R \to \infty} \int_{C_R} e^{az} f(z) \, dz = 0 \)
3. \( a > 0 \), \( \lim_{R \to \infty} \int_{C_R} e^{-az} f(z) \, dz = 0 \)
4. \( a < 0 \), \( \lim_{R \to \infty} \int_{C_R} e^{-az} f(z) \, dz = 0 \)

Theorem 3: If \( f(z) \to 0 \) uniformly as \( |z| \to \infty \), then

\[ \lim_{R \to \infty} \int_{C_R} f(z) \, dz = 0 \]

where \( C_R \)

Theorem 3 is a special case of Theorem 4.