1 Question 1

If you ordered the bars in the same order as the nodes (starting with bar 1 between nodes 1 and 2),

\[ A = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 1 \\ -1 & 1 & 0 & 0 & 0 & 0 & 1 & -1 \end{pmatrix} \]

You’d still get credit if you used a different order.

There were many ways to give an independent set of solutions. Here’s one independent set:

- the entire truss moves to the right
- the entire truss moves up
- the truss rotates clockwise about the origin
- nodes 1 and 3 move out, nodes 2 and 4 move in

The vectors corresponding to those solutions would be the columns of this matrix:

\[ \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix} \]

2 Question 2

Multiplying the equation by \( v(x) \) and integrating from 0 to 1,

\[ \left[ -e^x \frac{du}{dx} v(x) \right]_0^1 + \int_0^1 e^x \frac{du}{dx} \frac{dv}{dx} \, dx = v(a) \]

If you drop the first term, you get the weak form. This first term is already zero at \( x = 0 \) because of the boundary conditions on \( u(x) \). To make it be zero at \( x = 1 \), you need to impose

\[ v(1) = 0 \]

which both test functions in this example satisfy.
Substituting $u(x) = U_1\phi_1(x) + U_2\phi_2(x)$ and both $v(x) = V_1(x)$ and $v(x) = V_2(x)$ into the weak form above gives a system of equations $KU = F$ where

\[
K = \begin{pmatrix}
\frac{e^a - 1}{1 - e^a} & \frac{1 - e^a}{a^2} \\
\frac{e^a - 1}{a^2} & \frac{e^a - 1 - e^a}{(1 - a)^2}
\end{pmatrix}
\]

\[
F = \begin{pmatrix}
0 \\
1
\end{pmatrix}
\]

$K_e$ is the same as $K$ above except with the integrals performed only over the interval 0 to $a$. The result is the same except the lower right entry is missing the second term.