18.100B : Fall 2010 : Section R2
Homework 2

Due Tuesday, September 21, 1pm

**Reading:** Tue Sept.14 : countability, Euclidean spaces, Rudin 1.32-38, 2.1-17  
Thu Sept.16 : metric spaces, Rudin 2.15-28  
**Notes:** We will call $N_r(p)$ in 2.18(a) an open ball, rather than a neighborhood (which has a different meaning in general topology). The notion of perfect set is not important for us.

1. (a) Explain in your own words the logic of a proof by contradiction.  
   (b) Show that the set $\mathbb{R}$ of real numbers is uncountable. (You can use Theorem 2.14 for inspiration, but be aware that $0.\overline{9} = 0.9999999999\ldots = 1$.)

2. (a) Let $I = \{[p,q] ; p \leq q, p, q \in \mathbb{Q}\}$ be the set of intervals in $\mathbb{R}$ with rational endpoints. Show that $I$ is countable.  
   (b) Let $P$ be the set of all subsets of $\mathbb{N}$. Show that $P$ is uncountable.  
   [**Hint:** for an alleged bijection $f : \mathbb{N} \rightarrow P$, consider the set $\overline{D} = \{n \in \mathbb{N} ; n \not\in f(n)\}$]

3. Exercise 7, p. 43 of Rudin.

4. Exercise 8, p. 43 of Rudin.

5. Exercise 9, p. 43 of Rudin.