Reading: Tue Nov. 2: differentiability, mean value theorem, Rudin 5.1-11
Thu Nov. 4: l’Hospital’s rule, Taylor’s theorem, Rudin 5.12-19

1. Assume that \( f : \mathbb{R} \rightarrow \mathbb{R} \) and that for some \( C > 0 \) and \( \alpha > 0 \) we have for any \( x, y \in \mathbb{R} \)
\[
|f(x) - f(y)| \leq C|x - y|^\alpha.
\]

(a) Prove that if \( \alpha > 1 \) then \( f \) is constant.

[Hint: What is the derivative of a constant function?]

(b) If \( \alpha \leq 1 \), is \( f \) necessarily differentiable?

2. Problem #2 page 114 in Rudin.

3. (a) Problem #7 page 114 in Rudin.

(b) Show that for any polynomial \( P(x) \)
\[
\lim_{x \to \infty} \frac{P(x)}{e^x} = 0 \quad \text{and} \quad \lim_{x \to \infty} \frac{\ln(x)}{P(x)} = 0.
\]

For the second limit (of course) assume that \( P(x) \) is not constant. You may also use your calculus knowledge of derivatives of polynomials, \( e^x \), and \( \ln(x) \).

4. (a) Show that \( \sin(x) \approx x \) is a good approximation for small \( x \) by using Taylor’s theorem to obtain
\[
|\sin(x) - x| \leq \frac{1}{6}|x|^3 \quad \forall x \in \mathbb{R}.
\]

(b) Use (a) to calculate the limit for different values of \( a \in \mathbb{R} \) and \( c > 0 \) of the function \( x^a \sin(|x|^{-c}) \) (from Rudin pg.115 #13) as \( x \to \infty \).

5. (a) Assume \( f : (0, 1) \rightarrow \mathbb{R} \) is differentiable and \( |f'(x)| \leq M \) for all \( x \in (0, 1] \). Define the sequence \( a_n = f(1/n) \) and prove that \( a_n \) converges.

(b) Problem #26 page 119 in Rudin.
18.100B Analysis I
Fall 2010

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