18.100C Lecture 16 Summary


**Theorem 16.1** (Cauchy convergence criterion). A sequence of functions $f_n : X \to \mathbb{R}$ is uniformly convergent if and only if the following holds. For every $\epsilon > 0$ there is an $N$ such that if $m, n \geq N$ then $|f_n(x) - f_m(x)| < \epsilon$ for all $x$.

Uniform convergence of series of functions.

**Corollary 16.2.** (Weierstrass criterion) Let $\sum_{n=0}^{\infty} f_n$ be a series of functions. Suppose that there are constants $M_n$ such that $|f_n(x)| \leq M_n$ for all $n, x$, and such that $\sum_{n=0}^{\infty} M_n$ converges. Then $\sum_{n=0}^{\infty} f_n$ converges uniformly.

**Corollary 16.3.** Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $\rho > 0$. Then that series converges uniformly on any interval $[-r, r]$ with $r < \rho$.

**Theorem 16.4.** If $(f_n)$ are continuous functions converging uniformly towards $f$, then $f$ is again continuous.

**Corollary 16.5.** Let $f(x) = \sum_{n=0}^{\infty} a_n x^n$ be a power series with radius of convergence $\rho > 0$. Then $f$ is continuous on $(-\rho, \rho)$.