Problem Set 9

Do AG §3.5/3, 4, 7, 8, and the following additional problem.

(Alternative proof of Fourier inversion on \( \mathbb{R} \) using Fourier inversion on \( \mathbb{R}/2L\mathbb{Z} \).)

a) For \( f \in C^\infty(\mathbb{R}) \) periodic of period \( 2L \) define

\[
c_n = \frac{1}{2L} \int_{-L}^{L} f(x) e^{-in\pi x/L} \, dx
\]

Show (by change of variables) that

\[
f(x) = \sum_{n=-\infty}^{\infty} c_n e^{i\pi nx/L}
\]

b) For \( g \in C^\infty_0(\mathbb{R}) \), i.e., \( g \) infinitely differentiable with compact support, define

\[
\hat{g}(\xi) = \int_{-\infty}^{\infty} g(x) e^{-ix\xi} \, dx
\]

Use part (a) and justify the passage to the limit as \( L \to \infty \) to prove that

\[
g(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{g}(\xi) e^{ix\xi} \, d\xi
\]

(You may use the fact that \( \hat{g} \in \mathcal{S} \), the Schwartz class.)