Homework 3; due Thursday, Oct. 3

1. Generalize t'Hooft's theorem to integrals over quaternionic Hermitian matrices.

2. Find the number of ways to glue an orientable surface of genus $g \geq 1$ from a $4g$-gon (the gluing must preserve orientation), and prove your answer.

Answer: (4g-1)!!/(2g+1).

3. Consider a random Hermitian matrix $A \in \mathfrak{h}_N$, distributed with Gaussian density $e^{-Tr(A^2)}dA$. Show that the most likely eigenvalues of A are the roots of the N-th Hermite polynomial H_N.

Hint.

(1) Write down the system of algebraic equations for the maximum of the density on eigenvalues.
(2) Introduce the polynomial $P(z) = \prod_i (z - \lambda_i)$, where λ_i are the most likely eigenvalues. Let $f = P'/P$. Compute $f' + f^2$ (look at the poles).
(3) Reduce the obtained Riccati equation for f to a second order linear differential equation for P. Show that this equation is the Hermite's equation, and deduce that $P = H_N/2^N$.