Introduce scalar product definition of self-adjoint, and use to prove:

- Eigenvalues of self-adjoint matrices are real.
- Eigenvectors of self-adjoint matrices corresponding to distinct eigenvalues are orthogonal.
- NxN self-adjoint matrices have N orthogonal eigenvectors. Use descending technique: reduce size of problem by one by showing the matrix keeps the hyperplane orthogonal to an eigenvector invariant. Then use:
- Any matrix has, at least, one eigenvalue. Follows from "any polynomial has, at least, one root."
- Scalar product not unique. Examples:
  - \(<u, v> = \sum_n w_n u_n^* v_n\), for some weights \(w_n > 0\). One application:
    - cost functions in optimization.
- Any self-adjoint matrix with positive eigenvalues gives rise to a scalar product: \(u^* A v\). Prove this.