**Def.** A proper coloring of a graph is a coloring of vertices with no monochromatic edges. A grid graph $G_{m,n}$ is a product of a $m$-path and a $n$-path.

1) Let $c(n)$ be the number of proper colorings of $G_{n,n}$ with 3 colors. Prove
   a) $c(n) > C (1 + \varepsilon)^n$ for some $C, \varepsilon > 0$;
   b) $\frac{\log c(n)}{n^2} \to \alpha$ as $n \to \infty$, for some $\alpha > 0$.

2) Denote by $N_k$ be the number of proper colorings of $G_{n,n}$ with $k$ colors. Approximate $N_k$ up to 10% when
   a) $n = 100$ and $k = 1,000,000$;
   b) $n = 100$ and $k = 1,000$.

3) Consider the set $S_k(n)$ of proper colorings of $G_{n,n}$ with $k$ colors. Prove that for every two colorings $\chi, \chi' \in S_k(n)$, one can go from $\chi$ to $\chi'$ by changing one color at a time, when
   a) $k = 5$;
   b) $k = 4$.

4) In Schur’s theorem, the proof we presented gives $n(r) < er!$. Find an exponential lower bound by an explicit construction.

5) Consider random graphs $H$ on $n$ vertices with $m = 2n$ edges (defined as subgraphs of a complete graph $K_n$). What is more likely: that $H$ is bipartite or not?

6) An acute decomposition of a polygon $P \subset \mathbb{R}^2$ is a subdivision of $P$ into acute triangles, such that there are no vertices lying on the interior edges (see Figure 1). Prove that an acute decomposition of $P$ always exist if
   a) $P$ is a triangle;
   b) $P$ is a convex polygon which has an inscribed circle;
   c) $P$ is any convex polygon.

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**Figure 1.** A valid acute decomposition of a square, and an invalid one.

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Please remember to write the name(s) of your collaborators (see collaboration policy).