(Still) Chromatic + Tutte Polynomial

Thm (Stanley)
\[ \chi_G(-1) = (-1)^n \cdot \# \text{acyclic orientations of } G \]
(whence \( \chi(G) \geq n! \))

Pf: By induction, \( \chi(G) \cdot \# \text{a.o. of } G = \# \text{a.o. of } G \cdot e \)

(b/c acyclic orientation of \( G \cdot e \) \( \Rightarrow \) can orient \( e \) at least one way to get \( \text{a.o. } G \), if both ways \( \Rightarrow \) a.o. for \( G/e \) ("cause if no orientation for \( e \Rightarrow \exists \) cycle already")

Tutte polynomial
\[ T_G(x,y) = T_{G-e}(x,y) + T_{G/e}(x,y) \]
where \( d \) is dichromatic

New definition of contraction by an edge ("work with me people")

If \( e \) is a bridge or loop, then
\[ T_G(x,y) = \chi_T(x,y) \quad \text{if } e \text{ is a bridge} \]
\[ T_G(x,y) = \chi_T(x,y) \quad \text{if } e \text{ is a loop} \]

And for empty graph \( T_0(x,y) = 1 \)

Thm \( T_G(x,y) = T_G(y,x) \text{ where } G^* \text{ dual of } G \)
if \( G \) is planar (it all makes perfect sense if you know matroid theory)
Proposition: $T_G(x,y)$ is well-defined

Let $G = (V, W) ightarrow H \subseteq G \ni H = (V, F) \ orall F \in E$

Definition $T_G(x,y) = \sum_{F \in E} \frac{(x-1)^{c(H) - c(F)}}{(y-1)^{n(H)}}$

where $c(H) = \#$ conn. comp. of $H$

$n(H) = |F| - |V| + c(H)$ "nullity"

note that this satisfies recursive def'n