Open problems are marked with \(\star\); hard (but feasible) problems are marked with \(\odot\).

1. (Radon, 1952) Every set of \(d + 2\) points in \(\mathbb{R}^d\) can be partitioned into two subsets such that their convex hulls intersect.

2. (Las Vergnas & Lovász, 1972) You are given a finite range space \((X, R)\) such that for every \(k \in \mathbb{N}\), the union of every \(k\) ranges contains at least \(k + 1\) elements. Show that there is a 2-coloring of \(X\) such that no range is monochromatic.

3. (a) (Spencer, 1985) Given a range space \((X, R)\) with \(|X| = |R| = n\), show that its discrepancy is \(\text{disc}(R) = O(\sqrt{n})\).

   (b) (Beck, 1981) Given the range space \((X, R)\) where \(X = \{1, 2, \ldots, n\}\) and \(R\) is the set of all arithmetic progressions over \(X\), prove that \(\text{disc}(R) = O(n^{1/4} \text{ polylog } n)\). \(\star\star\)

4. (a) Every set of \(n\) disjoint unit disks in the plane has a BSP of size \(O(n)\).

   (b) Every set of \(n\) disjoint disks in the plane has a BSP of size \(O(n)\).

   (c) (de Berg, 2000) For every \(d \in \mathbb{N}\), every set of \(n\) disjoint fat objects in \(\mathbb{R}^d\) has a BSP of size \(O(n)\). (An object is fat if the ratio of the radii of the smallest enclosing and largest inscribed balls is bounded by a constant.)

5. (a) (Patterson & Yao, 1990) Every set of \(n\) pairwise disjoint axis-parallel lines in \(\mathbb{R}^3\) has a BSP of size \(O(n^{3/2})\). Find \(n\) disjoint axis-parallel lines in \(\mathbb{R}^d\) such that any BSP for them has size \(\Omega(n^{3/2})\).

   (c) The BSP-complexity of \(n\) disjoint axis-aligned rectangles in \(\mathbb{R}^3\) is \(\Theta(n^{3/2})\).

   (d) (Dumitrescu, Mitchell, and Sharir, 2004) The BSP-complexity of \(n\) disjoint axis-aligned 2-flats (i.e., 2-dimensional rectangles) in \(\mathbb{R}^4\) is \(\Theta(n^{5/3})\). \(\star\)

   (e) The BSP-complexity of \(n\) disjoint axis-parallel lines in \(\mathbb{R}^d\) is \(\Theta(n^{3/2})\).

   (f) What is the BSP-complexity of \(n\) disjoint axis-aligned 3-flats in \(\mathbb{R}^6\)? \(\star\)

6. (a) (Pach & Pinchasi, 2005) Every set of \(n\) pairwise disjoint line segments in the plane has a subset of \(\ell = \Omega(n^{1/3})\) segments that can be completed to a noncrossing simple path of \(2\ell\) vertices by adding \(\ell - 1\) new line segments between their endpoints.

   (b) There is a set of \(n\) pairwise disjoint line segments in the plane such that no subset of \(\ell = 2 \lceil \sqrt{2n} \rceil + 1\) segments can be completed to a noncrossing simple path this way.

7. (Aichholzer et al., 2004) The number of pointed pseudo-triangulations is minimized for point sets in convex position.

8. (Kettner et al., 2003) Every finite point set in general position in the plane has a pointed pseudo-triangulation

   (a) whose maximum vertex degree is five;

   (b) whose maximum face degree is four.