Hints for Problem Set 3

• **Problem 1e**
  Given a matrix $M$, how can you make a new matrix $N$ such that the largest eigenvalue of $N$ is related to the smallest eigenvalue of $M$?

• **Problem 1f**
  Try to generalize the idea from 1e. Given a matrix $M$, how can you create a new matrix $N$ so that the biggest eigenvalue of $N$ is related to the eigenvalue of $M$ closest to $\lambda$?

• **Problem 2b**
  If I have a polynomial $p(x)$ in one variable and I apply it to a matrix with the given eigenvalue bounds, I get a new matrix $N$. What can I say about its eigenvalues? What properties of $p$ will guarantee that $N$ still has the same eigenvector with eigenvalue 1 but that every vector orthogonal to it gets shrunk significantly in norm?

• **Problem 2c**
  Look really carefully at the description of conjugate gradient in Shewchuk's article. I claim that we've already computed everything I'm asking you for.

• **Problem 2e**
  For any polynomial $p$ of degree $t-1$, show that there is a vector $v$ in $\mathcal{K}(A,x,t)$ such that $v=p(A)x$. Argue that for a random $x$, there is some polynomial $p$ for which the corresponding $v$ gives a good Rayleigh quotient, and thus a good estimate for the biggest and smallest eigenvalues of $A$.

• **Problem 3**
  Make a new $2n \times 2n$ matrix $A$ and a vector $q$, and solve the linear system:
  \[ Ax = q \]
  $q$ will be the vector $[b; -b]$.
  $A$ will be made (somehow) by combining (somehow—maybe you’ll need to add them to each other, etc.) the following pieces:

  - $D =$ the diagonal of $M$
  - $P =$ the matrix of positive entries of $M$ (with everything else set to zero)
  - $N =$ the matrix of negative entries of $M$ (with everything else set to zero)

• **Problem 4d**
  You have to show how to route the edges of $G$ over those of $H'$ with low maximum total stretch, so that you can apply part (a). Part (c) tells you how to route the edges that are internal to a given $G_i$. Use this to get a bound on the contribution of these edges to the total stretch.

  You then just need to figure out how to route the edges that cross from one $G_i$ to some other $G_j$. Route these edges internally in $G_i$, across the bridge edge we’ve added, and then internally in $G_j$. Bound the contribution of these edges.