18.701 Problem Set 8

This assignment is due Monday, November 8

1. Chapter 6, Exercise 11.1. *(operations of $S_3$ on set of 4)*

2. Chapter 7, Exercise 5.12. *(class equations of $S_6$ and $A_6$)*

3. Chapter 7, Exercise 5.11a,b. *(class equations of $A_4$ and $A_5$)*

4. Chapter 7, Exercise 2.15 *(expanded)*
   
   (a) Let $F = \mathbb{F}_3$ and let $G = SL_2(F)$. Determine the centralizers and the orders of the conjugacy classes of the elements
   
   \[
   \begin{pmatrix}
   1 & 1 \\
   1 & 1 \\
   \end{pmatrix}
   \quad \text{and} \quad
   \begin{pmatrix}
   1 & -1 \\
   1 & 1 \\
   \end{pmatrix}.
   \]

   (b) By considering the center of $G$, prove that $G$ contains no conjugacy class of order 8 or 12.

   (c) The vector space $F^2$ contains four subspaces of dimension 1, and $G$ operates on the set of these subspaces. Determine the kernel and image of the corresponding permutation representation $\varphi : G \to S_4$.

   (d) Verify the class equation (7.2.10) of $G$.

5. Chapter 6, Exercise M.4. *(hypercube)*