Problem 1 (20 points)

Note: for reference material, consult the laboratory write-up on elastic-plastic beam bending

Consider the square cross-section beam shown, of dimensions $h$ by $h$, subject to “diamond-orientation” bending in the plane shown (neutral axis: plane $y = 0$). The beam can be considered to be composed of an elastic/perfectly-plastic material having Young’s modulus $E$, and tensile yield strength $\sigma_y$.

1. Using the standard assumptions of engineering beam theory, evaluate the magnitude of applied moment, $M_y$, just sufficient to bring the most highly-stressed region to the verge of yielding. Express your answer in terms of $h$ and material properties, as appropriate. (Aside: are you “surprised” by the value you got for $I = \int y^2 \, dA$ in this orientation?)

2. If the applied curvature is increased to very large values, the elastic/plastic boundaries (tension and compression sides) in this geometry, like those in the bending of rectangular cross-sections studied earlier, will move inward, toward the neutral axis. At “infinite” curvature, the boundaries will reach opposite sides of the $y = 0$ surface, resulting in tensile yielding stress values of magnitude $\sigma_y$ in one “triangle” half of the cross-section, and compressive yielding stress values of magnitude $-\sigma_y$ in the other triangular half of the cross-section. At this point, the bending moment carried by the cross-section reaches a limiting value, $M_L$. Evaluate $M_L$ for this section.

3. Using your answers to the two previous questions, evaluate the ratio $M_L/M_y$ for bending of this section. How does this value compare with the ratio for bending of this same cross-section, but on rotated axes, so that the cross-section appears as a square? (Our usual orientation for bending.)

4. Compare $M_y$ for the “diamond” cross-section with the corresponding $M_y$ for the square orientation. What is the ratio of these first-yield bending moments? Explain why they differ in the way that they do. Evaluate the same ratio for the corresponding limit moments, and $M_L$, and comment on reasons why they differ. Which axes should be used for applying bending moments to a square section, and why?
5. Discuss the residual stress state when the diamond-orientation is unloaded to $M = 0$ immediately after being deformed to large curvature at $M = M_L$. How does this residual stress state compare or contrast with the state for unloading of the square orientation from its limit value of $M$? Can any negative moment be applied to the diamond cross-section after unloading from limit load, without causing further plasticity? Discuss

![Figure 1: Square cross-section of beam, oriented for bending along “diamond” orientation.](image)

Problem 2 (30 points)

A great deal of the mechanisms and phenomenology of the strengthening of metallic crystals can be summarized in the following phrase:

“Smaller is stronger . . .”

Discuss three specific examples of strengthening mechanisms, and explain how and why the aphorism “smaller is stronger” applies to each strengthening mechanism.
Problem 3 (30 points)

Standard cylindrical compression specimens have an initial height to diameter ratio of $H_0/D_0 = 2$. It is desired to conduct a compression test in a demonstration lab, and to compress the specimen to a final height of $H = H_0/2$.

From prior testing, it is known that the material has Young’s modulus $E = 200 \text{ GPa}$, Poisson ratio $\nu = 0.3$, and its plasticity can be well characterized by an initial value of tensile/compressive yield strength as $s_0 = 500 \text{ MPa}$, along with a constant hardening modulus, $h = 2 \text{ GPa}$, governing the evolution of uniaxial flow strength, $s$, with equivalent plastic strain, $\bar{\varepsilon}^p$, according to

$$\frac{ds}{d\bar{\varepsilon}^p} = h = \text{constant}. $$

In turn, this expression can be integrated to express the current value of strength, for any given value of $\bar{\varepsilon}^p \geq 0$, as

$$s(\bar{\varepsilon}^p) = s_0 + h \bar{\varepsilon}^p. $$

The load cell on the testing machine to be used for the compression test has a maximum load capacity of 100 $kN$.

You are asked to provide an answer to the following question:

“What is the largest allowed value of initial diameter in a compression specimen of this material ($D_{0(\text{max})}$) that can be safely compressed to half its initial height in the testing machine?”

In particular:

- (10 points) Explain why the elastic strain is not an important feature in answering this problem. That is, explain why, for this application, you may assume that the material is rigid/plastic, so that the total strains and strain rates are essentially equal to the plastic strains and strain rates, respectively.

- (20 Points) What is the largest diameter that can safely be used for the compression specimen, under the imposed conditions?

HINTS:

- Remember, for active yielding in uniaxial compression, the axial [true]stress, $\sigma$, is negative, so the yield criterion becomes $s = \bar{\sigma} = -\sigma$.

- For monotonic loading in compression, the plastic portion of the [true] axial strain, $\epsilon = \epsilon^{(p)}$, is negative, and is thus related to the equivalent plastic strain by $-\epsilon^{(p)} = -\epsilon = \bar{\varepsilon}^p$. 
Problem # 4 (20 points)

Long bars of an alloy steel are available in stock of rectangular cross-section, with [initial] thickness $t_0 = 25 \text{mm}$ and width $w_0 = 100 \text{mm}$. It is desired to use these bars as tensile-loaded truss members, and to be able to apply tensile loads up to $P_{\text{max}} = 1.1 \text{MN}$ without causing plastic yielding in the bars. The initial tensile yield strength of the steel is $\sigma_y = 350 \text{MPa}$.

- Can the as-received bars support a load of magnitude $P_{\text{max}} = 1.1 \text{MN}$ without yielding? How much tensile load can it support without yielding?

- It is known that the tensile flow strength, $s$, of this steel increases with equivalent tensile plastic strain, $\bar{\varepsilon}^p$, according to
  \[ s(\bar{\varepsilon}^p) = \sigma_y \left(1 + \frac{\bar{\varepsilon}^p}{c}\right)^N, \]
  where the strain hardening exponent is $N = 0.14$, and the constant $c = 0.01$. Someone suggests that it may be possible to cold-roll the bar stock to a new cross-sectional shape, of reduced thickness $t$, but essentially the same width, $w = w_0$, and in the process generate enough equivalent plastic strain and associated strain-hardening so that the rolled bar stock can be used as truss members that can support tensile loads up to $P_{\text{max}} = 1.1 \text{MN}$ without [further] plastic yielding, even though the rolling reduces the thickness and cross-sectional area of the bar. We will explore this possibility.

First note that the equivalent plastic strain increment, $d\bar{\varepsilon}^p$, can be expressed in terms of the cartesian components of the plastic strain increment tensor, $\varepsilon_{ij}^{(p)}$, by

\[ d\bar{\varepsilon}^p = \sqrt{\frac{2}{3} \sum_{i=1}^{3} \sum_{j=1}^{3} \varepsilon_{ij}^{(p)} \varepsilon_{ij}^{(p)}}. \]

Let the rolling direction (along the length of the bar) be cartesian direction number 1, let the through-thickness direction be 2, and let the breadth direction be 3. In the process of rolling, there is an incremental reduction in thickness, $dt < 0$, so that

\[ d\varepsilon_{22}^{(p)} = \frac{dt}{t} < 0. \]

As noted above, there is negligible transverse plastic straining in rolling, so $d\varepsilon_{33}^{(p)} = 0$. Assume further that rolling introduces no change in plastic shear strains (i.e., $d\varepsilon_{12}^{(p)} = d\varepsilon_{13}^{(p)} = d\varepsilon_{23}^{(p)} = 0$).

**Obtain an expression for $d\bar{\varepsilon}^p$ in terms of $t$ and $|dt|$, and show how this expression can be integrated to give**

\[ \bar{\varepsilon}^{(p)} = \frac{2}{\sqrt{3}} \ln \left(\frac{t_0}{t}\right). \]

HINT: something needs to be done about evaluating $d\varepsilon_{11}^{(p)}$...
• What is the maximum rolling-reduced bar thickness, \( t = t_{\text{max}} \), which gives a strain-hardened strength \( s \) and rolling-reduced thickness \( t = t_{\text{max}} \) combination such that the cold-rolled bar stock does, indeed, support tensile load \( P_{\text{mx}} = 1.1 \text{MN} \) without further yielding?

Note: this part of the problem may best be solved by performing a set of numerical evaluations, for different values of thickness, and finding out which \( t \)-value answers the question.