Cranking up the gain 😊

**Type 0 system (no disturbance)**

Steady-state error due to step input:

\[ e_R(\infty) = \frac{2}{2 + 0.3162K} \]

\[ e_R(\infty) \to 0 \text{ as } K \to \infty \]

\[ V_{ref}(s) + E(s) \quad K \quad \frac{0.3162}{s+2} \quad V(s) \]

**Type 1 system with disturbance**

Steady-state error due to step input:

\[ e_R(\infty) = 0 \]

Steady-state error due to step disturbance:

\[ e_D(\infty) = -\frac{1}{K} \]

\[ e_D(\infty) \to 0 \text{ as } K \to \infty \]

\[ V_{ref}(s) + E(s) \quad K \quad D(s) \quad \frac{0.3162}{s(s+2)} \quad X(s) \]
Cranking up the gain 😁

**Type 1 system (no disturbance)**

\[ \frac{X(s)}{V_{ref}(s)} = \frac{0.3162K}{s^2 + 2s + 0.3162K} \]

Closed-loop transfer function

Pole locations
\[ p_1 = -1 + \sqrt{1 - 0.3162K} \quad p_2 = -1 - \sqrt{1 - 0.3162K} \]

System becomes **underdamped** ⇒ step response **overshoots** if

\[ 1 - 0.3162K < 0 \iff K > 3.1626 \]
Cranking up the gain: poles and step response

Pole-Zero Map

$K$ increasing
$K > 3.1626$

$K$ increasing
$K < 3.1626$

Root Locus
Root locus for nonunity feedback systems

Caveat: $K > 0$

Closed-loop pole locations

$$1 + KG(s)H(s) = 0 \Rightarrow \left\{ \begin{array}{l} K = \frac{1}{|G(s)H(s)|}; \\ \angle KG(s)H(s) = (2n + 1)180^\circ. \end{array} \right.$$
Root locus terminology

- Asymptote
- Branches
- Real-axis segment
- Breakaway point
- Asymptote angle
- Asymptote real-axis intercept

Break-in point
Breakaway point

RL imaginary axis intercept

s-plane

Departure/Arrival angles

ζ = 0.45

Figure by MIT OpenCourseWare.
Root-locus sketching rules

• **Rule 1:** # branches = # poles
• **Rule 2:** symmetrical about the real axis
• **Rule 3:** real-axis segments are to the left of an *odd* number of real-axis finite poles/zeros

Let \( G(s) = \frac{N_G(s)}{D_G(s)} \), \( H(s) = \frac{N_H(s)}{D_H(s)} \).

\[ \Rightarrow \angle G(s)H(s) = \sum \angle(\text{poles}) - \sum \angle(\text{zeros}). \]

Recall angle condition for closed-loop pole:

\[ \angle KG(s)H(s) = (2n + 1)180^\circ. \]

Complex-pole/zero contributions: **cancel** because of symmetry
Real-pole/zero contributions: each is \( 0^\circ \) from the left, \( 180^\circ \) from the right; total contributions from right must be odd number of \( 190^\circ \)'s to satisfy angle condition.

Image removed due to copyright restrictions.

Root-locus sketching rules

• **Rule 4:** RL begins at poles, ends at zeros

\[ G(s) = \frac{N_G(s)}{D_G(s)}, \quad H(s) = \frac{N_H(s)}{D_H(s)}. \]

\[ \Rightarrow \quad \text{Closed–loop TF} = \frac{K N_G(s) D_H(s)}{D_G(s) D_H(s) + K N_G(s) N_H(s)}. \]

If \( K \to 0^+ \) (small gain limit)

Closed–loop TF \( \approx \frac{K N_G(s) D_H(s)}{D_G(s) D_H(s)} \Rightarrow \)

closed–loop denominator is *denominator* of \( G(s)H(s) \)
\( \Rightarrow \) closed–loop poles are the *poles* of \( G(s)H(s) \).

If \( K \to +\infty \) (large gain limit)

Closed–loop TF \( \approx \frac{K N_G(s) D_H(s)}{\epsilon + K N_G(s) N_H(s)} \Rightarrow \)

closed–loop denominator is *numerator* of \( G(s)H(s) \)
\( \Rightarrow \) closed–loop poles are the *zeros* of \( G(s)H(s) \).

**Example**

![s-plane diagram](Nise Figure 8.10)

Figure by MIT OpenCourseWare.
Please see the following selections from


Ch. 4, pp. 3-18