2.008 Quiz 2 Review
Quiz Topics

- **Casting:** $V/SA$ vs. $(V/SA)^2$
- **Forming:** Force, elastic v. plastic deformation.
- **Process:** shop, project, flow, cell.
- **Systems:** $L = \lambda w$
- **Quality:** $C_{pk}$, SPC.
- **Cost:** allocations
- **Layered Manufacturing**
- **MEMS** Look this over!
Photo Resists

Radiation

Mask

Positive Resist

Negative Resist

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Laser Cutting Problem

A laser cutter cuts its material by evaporating the material along the cut line. In terms of the following variables:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Variable</th>
<th>Units</th>
</tr>
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<tbody>
<tr>
<td>Mass</td>
<td>m</td>
<td>grams</td>
</tr>
<tr>
<td>Ambient Temperature</td>
<td>( t_o )</td>
<td>°K</td>
</tr>
<tr>
<td>Melting Temperature</td>
<td>( t_m )</td>
<td>°K</td>
</tr>
<tr>
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<td>( t_b )</td>
<td>°K</td>
</tr>
<tr>
<td>Solid specific heat capacity</td>
<td>( c_p )</td>
<td>( J(kG)^{-1} °K^{-1} )</td>
</tr>
<tr>
<td>Latent heat of fusion</td>
<td>( h_f )</td>
<td>( J(kG)^{-1} )</td>
</tr>
<tr>
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<td>( J(kG)^{-1} °K^{-1} )</td>
</tr>
<tr>
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<td>( h_v )</td>
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</tr>
<tr>
<td>Density</td>
<td>( \rho )</td>
<td>( kg/m^3 )</td>
</tr>
<tr>
<td>Material thickness</td>
<td>( h )</td>
<td>m</td>
</tr>
<tr>
<td>Radius of laser spot</td>
<td>( r )</td>
<td>m</td>
</tr>
<tr>
<td>Power of the laser</td>
<td>( P )</td>
<td>W</td>
</tr>
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Write a formula which describes how much energy per gram is required to vaporize a material.
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$$E_{\text{vaporize}} = m((t_m - t_o)c_p + h_f + (t_b - t_m)c_l + h_v)$$
Laser Cutting

The laser beam is focused onto a circle of radius \( r \). Assume that all of the energy of the laser is all delivered to this circle, and that all of the energy is used in the cutting process (ie, there is no energy loss due to spectral absorption). Let \( \rho \) be the density of the material, and let \( h \) be the material thickness. Write an equation governing speed of cutting to the power of the laser, \( P \).
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The laser’s energy, \( P \), is delivered uniformly over an area of \( \pi r^2 \). In order to cut through this material, a volume of \( \pi r^2 h \text{ mm}^3 \) must be evaporated. This volume will require

\[
e = \frac{\pi r^2 h E_{\text{vaporize}}}{\rho} \text{ Joules}
\]

of energy to vaporize. If \( P \) represents watts of laser power available, then \( P/e \) seconds are required for the laser to cut through the material. Since the laser can safely move \( 2r \) during this interval, the cutting velocity can be

\[
v_c = 2re/P
\]

This assumes that none of the heat is lost to diffusion. In fact, if \( v_c \) is greater than the rate of diffusion, this is assumption is valid.
Limiting Factors

What physical quantity limits how fast you can operate the following processes:

- Turning
- Sand casting
- Injection molding
- Milling
- Forging
- Die casting
- Thermoforming
- MIG Welding
- Friction Welding
Limiting Factors

What physical quantity limits how fast you can operate the following processes:

- Turning Delivery of power to cutting tool
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What physical quantity limits how fast you can operate the following processes:

- Turning: Delivery of power to cutting tool
- Sand casting: Diffusion of heat
- Injection molding: Diffusion of heat
- Milling: Delivery of power to cutting tool
- Forging: Force required to plastically deform metal
- Die casting
- Thermoforming
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- **Thermoforming** Heating of material to glassy state
- **MIG Welding**
- **Friction Welding**
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- Die casting: Conductance of heat
- Thermoforming: Heating of material to glassy state
- MIG Welding: Delivery of current to melt weld area
- Friction Welding: Generating enough heat via friction
Why are nucleation agents added to a metal casting?
Short Answers

Why are nucleation agents added to a metal casting?

Nucleation agents promote equaxis crystal growth.

What happens to grain size as the cooling time increases?
Short Answers

- Why are nucleation agents added to a metal casting?

  Nucleation agents promote equaxis crystal growth.

- What happens to grain size as the cooling time increases?

  Grain size decreases as cooling time increases.
You have been assigned to set up a manufacturing process for making license plates. You will be responsible for setting up the line and ensuring it meets rate, quality and cost goals.

Which (if any) of these materials (A and/or B and/or C) can not be used to form the license plate? Justify your answer with a short explanation.
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Which (if any) of these materials (A and/or B and/or C) can not be used to form the license plate? Justify your answer with a short explanation.

Solution: Material C is not usable because it has no plastic deformation before failing – it is perfectly elastic.
In the following process:

- Blank plate shape
- Form outer ring
- Form letter + numbers
- Punch four holes
- Coat Paint Detail
- Inspect

the maximum strain ($\epsilon_m$) in the license plate falls within one of three categories:

- a: $0 < \epsilon_m < \epsilon_1$
- b: $\epsilon_1 < \epsilon_m < \epsilon_2$
- c: $\epsilon_m = \epsilon_2$

Indicate (using a,b,c) the category which describes the maximum strain in each of the following operations:

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<tr>
<th>Operation 1</th>
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<th>Operation 4</th>
</tr>
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<tbody>
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<td></td>
<td></td>
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![Process Diagram]

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<td></td>
<td></td>
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Reasons

- Material is cut.
In the following process:

Material is cut. 

Material is plastically deformed to accept geometry.

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<th>Operation 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>B</td>
<td>B</td>
<td>had to be blanked out</td>
</tr>
</tbody>
</table>

Reasons

- ⬤ Material is cut.
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In the following process:

1. Blank plate shape
2. Form outer ring
3. Form letter + numbers
4. Punch four holes
5. Coat Paint Detail
6. Inspect

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Reasons

- Material is cut.
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- Material is plastically deformed to accept geometry.
- Material is cut.
Given a constant stamping speed (speed of the die as it closes) and a specific license plate geometry, which of the usable materials (among A, B, or C) would you choose if you wanted to minimize the power/energy required to make the license plate? Justify your answer using words and basic physics/equations.
Given a constant stamping speed (speed of the die as it closes) and a specific license plate geometry, which of the usable materials (among A, B, or C) would you choose if you wanted to minimize the power/energy required to make the license plate? Justify your answer using words and basic physics/equations.

Material B minimizes the power/energy requirements. For a constant speed, the power is proportional to the energy, and energy is proportional to \( \sigma(\varepsilon^2) \), and so the lower stress curve represents lower required energy.
Which of the usable materials (among A, B, or C) would you choose to minimize the amount of spring back? Justify your answer.
Which of the usable materials (among A, B, or C) would you choose to minimize the amount of spring back? Justify your answer.

Material A minimizes spring back; following the path of elastic modulus back from the ultimate forming strain results in material B recovering more than material A, as shown below.
Given the following processing times (per part) for the manufacturing line:

- Blank plate shape: 20s
- Form outer ring: 20s
- Form letter + numbers: 20s
- Punch four holes: 20s
- Coat Paint Detail: 40s
- Inspect: 20s

calculate the production rate in parts/hour.
Forming [Q2 ’02]

Given the following processing times (per part) for the manufacturing line:

<table>
<thead>
<tr>
<th>Process</th>
<th>Time</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blank plate shape</td>
<td>20s</td>
<td>180p/h</td>
</tr>
<tr>
<td>Form outer ring</td>
<td>20s</td>
<td>180p/h</td>
</tr>
<tr>
<td>Form letter + numbers</td>
<td>20s</td>
<td>180p/h</td>
</tr>
<tr>
<td>Punch four holes</td>
<td>20s</td>
<td>180p/h</td>
</tr>
<tr>
<td>Coat Paint Detail</td>
<td>40s</td>
<td>90p/h</td>
</tr>
<tr>
<td>Inspect</td>
<td>20s</td>
<td>180p/h</td>
</tr>
</tbody>
</table>

The painting step limits the process:

\[
60 \text{ s/m} / 40 \text{ s/part} \times 60 \text{ m/hr} = 90 \text{ parts/hr}
\]
Now consider the following MTTF/MTTR data for each machine:

<table>
<thead>
<tr>
<th>Process</th>
<th>MTTF (h)</th>
<th>MTTR (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blank plate shape</td>
<td>4.5</td>
<td>30</td>
</tr>
<tr>
<td>Form outer ring</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>Form letter + numbers</td>
<td>3</td>
<td>20</td>
</tr>
<tr>
<td>Punch holes</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>Coat Paint Detail</td>
<td>45</td>
<td>5</td>
</tr>
<tr>
<td>Inspect</td>
<td>4.75</td>
<td>15</td>
</tr>
</tbody>
</table>

Calculate the upper bound on production rate in parts/hour.
Now consider the following MTTF/MTTR data for each machine:

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Calculate the upper bound on production rate in parts/hour.

Compute MTTF/(MTTF+MTTR) for each process. Then multiply this availability factor by each of the production rates. The punch operation has \( \frac{40}{40+60} = 0.40 \).

\[
Rate_{punch} = (0.40) \left( \frac{60\text{sec/min}}{20\text{sec/part}} \right) \times 60\text{min/hr} = 72 \text{ parts/min}
\]
Now consider the following MTTF/MTTR data for each machine:

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<tr>
<td>Blank plate shape</td>
<td>4.5h</td>
<td>30m</td>
<td>0.90</td>
</tr>
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<td>0.90</td>
</tr>
<tr>
<td>Inspect</td>
<td>4.75h</td>
<td>15m</td>
<td>0.95</td>
</tr>
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</table>

Calculate the upper bound on production rate in parts/hour.

Compute $\frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}}$ for each process. Then multiply this availability factor by each of the production rates. The punch operation has $\frac{40}{40 + 60} = 0.40$.

$\text{Rate}_{\text{punch}} = (0.40)(\frac{60 \text{sec/min}}{20 \text{sec/part}} \ast 60 \text{min/hr}) = 72 \text{ parts/min}$

Would adding a single buffer help? If so, where?
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$$Rate_{punch} = (0.40)(60\text{sec/min}/20\text{sec/part} \times 60\text{min/hr}) = 72 \text{ parts/min}$$

Would adding a single buffer help? If so, where?

Add a buffer after the punch to offset machine downtime.
Another option for increasing the production rate is adding a machine. Option 1 is to rent a manual machine, which will require an additional person to run; Option 2 is purchasing a more expensive automatic machine that can be run by the existing operator.

Profit: $1.50/part
Shift time: 2000 hours/year
Labor rate: $20/hour
Labor overhead: 20%
Manual machine: $3,000/month
Automatic machine: $100,000

Assuming you want the decision to produce the lowest cost after one year, which option is best?
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Assuming you want the decision to produce the lowest cost after one year, which option is best?

Option 1 cost = ([3000 $/mo] + [20 $/hr] * [1.2] * [167 hrs/mo]) * [12 months] = $84096
Option 2 cost = $100,000
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How much time is required before the options are equal from a financial perspective?
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\[3000 \text{$/mo} + 20 \text{/hr} \times 1.2 \times 167 \text{hrs/mo} \times 12 \text{months} = 84096\]

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How much time is required before the options are equal from a financial perspective?

\[3000 \text{$/mo} + 20 \text{/hr} \times 1.2 \times 167 \text{hrs/mo} \times m \text{ months} = 100,000 \rightarrow m = 14.3 \text{ mo}\]
In addition to rate, quality is an issue. Assuming there are two painting stations, the two stations do not provide the same quality. The main factor for quality is alignment of the paint layer with the embossed features in the sheet metal. Both the average value for alignment and distribution of the alignment are important to overall quality. The two stations in the manufacturing line differ in both mean value and standard deviation.

Machine A: \( \bar{x} = 0.80\text{mm}, \sigma = 0.15\text{ mm} \)
Machine B: \( \bar{x} = 0.00\text{ mm}, \sigma = 0.48\text{ mm} \)

The specifications for painting the license plates call for alignment of 0 \( \pm \) 1.0mm. Calculate \( C_p \) and \( C_{pk} \) for each machine.
In addition to rate, quality is an issue. Assuming there are two painting stations, the two stations do not provide the same quality. The main factor for quality is alignment of the paint layer with the embossed features in the sheet metal. Both the average value for alignment and distribution of the alignment are important to overall quality. The two stations in the manufacturing line differ in both mean value and standard deviation.

Machine A: \( \bar{x} = 0.80 \text{mm}, \sigma = 0.15 \text{mm} \)

Machine B: \( \bar{x} = 0.00 \text{mm}, \sigma = 0.48 \text{mm} \)

The specifications for painting the license plates call for alignment of \( 0 \pm 1.0 \text{mm} \). Calculate \( C_p \) and \( C_{pk} \) for each machine.

A: \( C_p = \frac{\text{Range}}{6\sigma} = \frac{[2\text{mm}]}{0.9\text{mm}} = 2.22 \)

B: \( C_p = \frac{[2\text{mm}]}{2.88\text{mm}} = 0.69 \)
Calculate the resulting yield (percentage of parts within the specifications) for each machine.

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</tbody>
</table>
Calculate the resulting yield (percentage of parts within the specifications) for each machine.

Use the charts for Z:

A

\[
Z_{\text{min}} = \frac{1 - 0.8}{0.15} = 1.33 \rightarrow [P = 0.9082], (1 - P) = 0.0918
\]

\[
Z_{\text{max}} = \frac{1 + 0.8}{0.15} = 12.0 \rightarrow [P \approx 1.000], (1 - P) = 0
\]

Yield = 1 - 0.0918 - 0.0000 = 0.9082 = 90.8

B

\[
Z_{\text{min}} = \frac{1 - 0}{0.48} = 2.08 \rightarrow P = 0.9812, (1 - P) = 0.0188
\]

\[
Z_{\text{max}} = \frac{1 - 0}{0.48} = 2.08 \rightarrow P = 0.9812, (1 - P) = 0.0188
\]

Yield = 1 - 0.0188 - 0.0188 = 0.9624 = 96.2
Let $C(x) = 2250(0.015 - 0.001)x^2 + 6000x = 225x^2 - 750x + 50625$.

It pays to buy 1.66 units of consulting.

Homework 7-3

Your enterprise is using a process which produces parts whose length have a mean 2.5in and standard deviation 0.015in. You plan to make 100,000 of these parts. You have determined by taking into account the rework cost and the loss of customer satisfaction that the quality loss function for the process is:

$$L = 2250\sigma^2$$

where $\sigma$ is the standard deviation of length of the part and $L$ is the average quality lost in dollars per part.

A very smart but somewhat expensive Harvard graduate has offered you her expert advice on your process. For every 0.001in reduction in the standard deviation of the length of the part, she will charge $6000.

How much of her advice do you buy? (You can buy fractional amounts of her consulting).
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\[ L = 2250\sigma^2 \]

where \( \sigma \) is the standard deviation of length of the part and \( L \) is the average quality lost in dollars per part.

A very smart but somewhat expensive Harvard graduate has offered you her expert advice on your process. For every 0.001in reduction in the standard deviation of the length of the part, she will charge $6000.

How much of her advice do you buy? (You can buy fractional amounts of her consulting).

Let \( C(x) \) be the cost function. Minimize by taking derivatives:

\[
C(x) = (10^5)2250(0.015 - 0.001x)^2 + 6000x
\]

\[
= 225x^2 - 750x + 50625
\]

\[
C'(x) = 450x - 750
\]

\[ x = 1.66 \]

It pays to buy 1.66 units of consulting.