Problem 1  (20 points)

Servomotors make the flywheel spin at a constant rate $\omega_2$, and also impose a vertical rotation rate $\omega_1$ that is a function of time (see figure below). The center of mass of the flywheel is located on the $z$ axis, and the centroids of inertia are $I_1$ about the spin axis and $I_2$ transverse to that axis.

(a) Derive the equations of motion for the system.

(b) Determine the external torques necessary to maintain the above motion.
Problem 2  (20 points)

A solid uniform cylinder of mass $m$ and radius $R$ is placed on top of a fixed cylinder of the same radius, and it is slightly tipped, as shown in the figure. Find the value of the angle $\theta$ at which sliding begins as a function of the static friction coefficient $\mu$. 

![Diagram of a cylinder on top of another cylinder with an angle $\theta$.](image)