Problem Set No. 3

Out: Thursday, March 15, 2007
Due: Thursday, April 5, 2007 in class

Problem 1

Consider the Van der Pol oscillator

\[ \ddot{x} + \epsilon (x^2 - 1) \dot{x} + x = 0. \]

As discussed in class, this nonlinear vibratory system has a stable limit cycle which for \( 0 < \epsilon \ll 1 \) may be approximated as

\[ x = 2 \cos t + O(\epsilon). \]

Your job is to calculate the frequency \( \omega \) of this limit cycle correct to \( O(\epsilon^2) \); i.e., to find \( \omega_2 \) in the expansion

\[ \omega = 1 + \omega_2 \epsilon^2 + \cdots. \]

Problem 2

A nonlinear mass-spring vibratory system in the presence of dry friction and subject to harmonic excitation is governed by the following dimensionless equation

\[ \ddot{x} + 2 \zeta \text{sgn} \dot{x} + x + \epsilon x^2 - F \cos \frac{\Omega}{\omega_n} t \]

where \( \frac{\Omega}{\omega_n} \approx 1 \) and \( F = O(1) \). Assume light damping \( (\zeta \ll 1) \) and weak nonlinearity \( (\epsilon \ll 1) \).
Find the appropriate scaling of the small parameter $\zeta$ in terms of $\epsilon$ so that light damping and weak nonlinear effects are equally important near resonance. What is the width and height of the resonance peak in terms of $\epsilon$?

Note: the spring nonlinearity is quadratic.

Problem 3

Consider a Van der Pol oscillator with a cubic nonlinear spring under harmonic forcing:

$$\ddot{x} - \epsilon(1 - x^2)\dot{x} + x + \epsilon x^3 - F \cos \frac{\Omega}{\omega_n} t.$$

Assuming $0 < \epsilon \ll 1$, analyze the response for

(a) $F = O(\epsilon)$ and $\frac{\Omega}{\omega_n} \approx 1$

(b) $F = O(1)$ and $\frac{\Omega}{\omega_n} \approx 3$

(c) $F = O(1)$ and $\frac{\Omega}{\omega_n} \approx \frac{1}{3}$.