Today’s goal

• Root Locus examples and how to apply the rules
  – single pole
  – single pole with one zero
  – two real poles
  – two real poles with one zero
  – three real poles
  – three real poles with one zero

• Extracting useful information from the Root Locus
  – transient response parameters
  – limit gain for stability
Root Locus definition

- Root Locus is the locus on the complex plane of **closed**-loop poles as the feedback gain is varied from 0 to $\infty$.

\[
\left( \frac{X(s)}{Y(s)} \right)_{OL} = \frac{1}{s + 2}
\]

\[
\left( \frac{X(s)}{Y(s)} \right)_{CL} = \frac{K}{s + 2 + K}
\]

As $K$ varies from 0 to $\infty$ . . .
Root-locus sketching rules

- **Rule 1:** # branches = # poles
- **Rule 2:** always symmetric with respect to the real axis
- **Rule 3:** real-axis segments are to the left of an *odd* number of real-axis finite poles/zeros
Root-locus sketching rules

- **Rule 4:** begins at poles, ends at zeros

\[
\left( \frac{X(s)}{Y(s)} \right)_{CL} = \frac{K}{s + (K + 2)}
\]

\[
\left( \text{closed-loop pole} \right) = -(K + 2) \to -\infty, \quad \text{as} \quad K \to \infty
\]

We say that this TF has a "zero at infinity"

\[
\left( \frac{X(s)}{Y(s)} \right)_{OL} = \frac{1}{s + 2}
\]

\[
\left( \text{closed-loop pole} \right) = \frac{-5K + 2}{K + 1} \to -5, \quad \text{as} \quad K \to \infty
\]
Root Locus sketching rules

- **Rule 5:** Real-axis intercept and angle of asymptote

\[
\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\# \text{finite poles} - \# \sum \text{finite zeros}}
\]

\[
\theta_a = \frac{(2m + 1) \pi}{\# \text{finite poles} - \# \sum \text{finite zeros}}
\]

\[G(s) = \frac{1}{(s + 1)(s + 3)}
\]

- Two branches (Rule 1)
- Symmetric (Rule 2)
- On real axis to the left of the first pole at -1 (Rule 3)
- Two zeros at infinity (Rule 4)

\[G(s) = \frac{1}{(s + 1)(s + 2)(s + 3)}
\]

- Three branches (Rule 1)
- Symmetric (Rule 2)
- On real axis to the left of the first pole at -1 and the third pole at -3 (Rule 3)
- Three zeros at infinity (Rule 4)
Root Locus sketching rules

- **Rule 6**: Real axis breakaway and break-in points $\sigma_b$

\[
\sum_n \frac{1}{\sigma_b - z_n} = \sum_q \frac{1}{\sigma_b - p_q}
\]

\[G(s) = \frac{1}{(s + 1)(s + 2)}\]

- Two branches (Rule 1)
- Symmetric (Rule 2)
- On real axis to the left of the first pole at -1 (Rule 3)
- Two zeros at infinity (Rule 4)

\[G(s) = \frac{1}{(s + 1)(s + 2)(s + 3)}\]

- Three branches (Rule 1)
- Symmetric (Rule 2)
- On real axis to the left of the first pole at -1 and the third pole at -3 (Rule 3)
- Three zeros at infinity (Rule 4)
Root Locus sketching rules

- **Rule 7**: Imaginary axis crossings

\[
\text{Solve } KG(j\omega) = -1
\]
What else is the Root Locus telling us

- Gain = product of distances to the poles

\[
G(s) = \frac{1}{(s + 1)(s + 2)}
\]

\[
G(s) = \frac{1}{(s + 1)(s + 2)(s + 3)}
\]

\[K = 60\]
The zeros are “pulling” the Root Locus

- Because of Rule 4
- Therefore, adding a zero makes the response
  - faster
  - stable

\[ G(s) = \frac{s + 5}{(s + 1)(s + 3)} \]
Practice 1: Sketch the Root Locus

(a)  

(b)  

(c)  

(d)  

(e)  

(f)  

Nise Figure P8.2

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Practice 2:
Are these Root Loci valid? If not, correct them

Nise Figure P8.1

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