Massachusetts Institute of Technology
Department of Mechanical Engineering

2.12 Introduction to Robotics
Problem Set No.1
Out: September 14, 2005   Due: September 21, 2005

Problem 1

The torque-speed characteristics of a DC motor have been determined, as shown in the figure below. Note that u is the voltage applied to the motor armature, τ_m and ω_m are, respectively, the output torque and angular velocity of the motor shaft. From this plot,

(a) Obtain the motor constant, torque constant, and armature resistance of the motor.
(b) When the motor is rotating at 100 radians per second, what is the back emf voltage induced at the armature?
(c) When the motor is producing 2×10^{-3} Nm of torque, how much power is dissipated as heat at the motor windings?
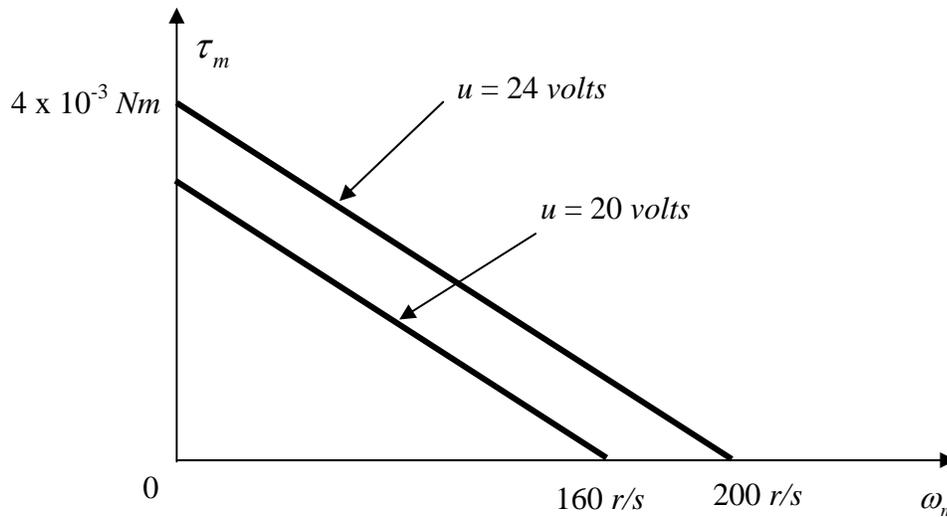

Figure 1 Torque-speed characteristics of a DC motor

Problem 2

The DC motor in Problem 1 is now used for driving a single axis robot arm with a gear reducer. See the figure below. The inertia of the motor rotor including the shaft and the pinion gear is $I_m = 1.0 \times 10^{-4}\ kgm^2$, while the arm inertia about the joint axis including the joint axis and the large gear is $I_a = 1.6 \times 10^{-1}\ kgm^2$. Obtain the gear ratio that maximizes the angular acceleration of the joint axis, when the angular velocity is almost zero. Ignore gravity and viscous damping. Also obtain the maximum angular acceleration when the maximum armature voltage is 30 *volts*.
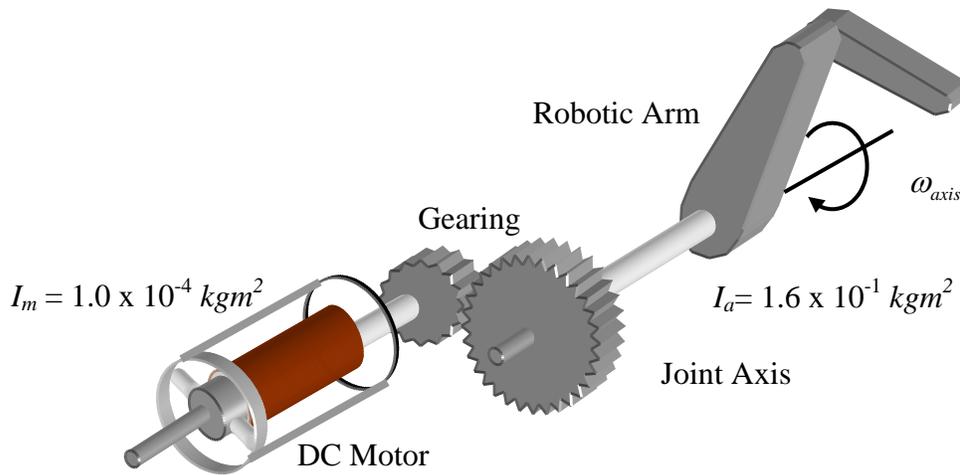
Figure 2 Single axis robot arm with DC motor and gearing

Problem 3

Consider a single axis robot control system with an optical shaft encoder measuring both position and velocity of the joint axis. The transfer function from input armature voltage u to the joint angle θ is given by

$$G(s) = \frac{\theta(s)}{u(s)} = \frac{1}{s(a_1 s + a_2)}$$

where parameters are $a_1 = 0.16$, $a_2 = 0.53$. Answer the following questions.

(a) Consider the position and velocity feedback control, as shown in the first figure below. Determine the position feedback gain k_p and the velocity feedback gain k_v so that the 0-100% rise time is 0.4 sec and that the maximum overshoot is 5 %.

(b) Consider the integral control along with the position and velocity feedbacks, as shown in the second figure below. Sketch a root locus for appropriate values of the velocity feedback gain k_v and the integral gain k_I, and discuss stability, settling time, and steady-state error. [I believe that you have learned these in 2.004. If not, please let me know.]
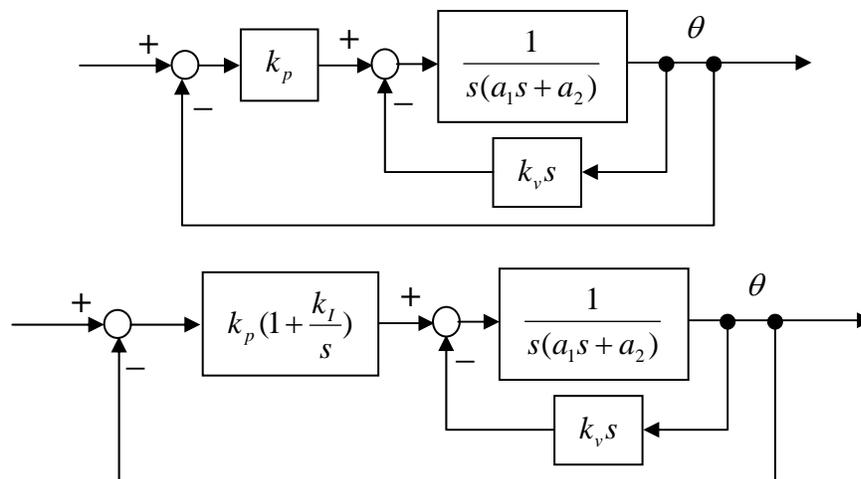

Figure 3 Block diagrams

Problem 4

A DC motor connected to a switching power transistor is shown in Figure 4-a. Answer the following questions.

a). The torque constant of the motor is 5.0×10^{-2} Nm/A. What is the voltage across the motor armature as the motor rotates at 100 rad/s with a zero torque load?

b). Figure 1-b shows profiles of the transistor voltage and current, V_{ce} and I_c, when the transistor turns on and off. The maximum voltage is 10 volts, and the maximum current is 0.5 A. It takes 5 μs for the transistor to turn on, and 10 μs to turn-off. During the turn-on and turn-off transition periods both voltage and current vary linearly, as shown in the figure. How much heat (cal) is generated at the transistor in one second when this transistor is used for a uni-polar PWM amplifier of 10 kHz PWM frequency? Note that the mechanical equivalent of heat quantity is 4.2 joule/cal.
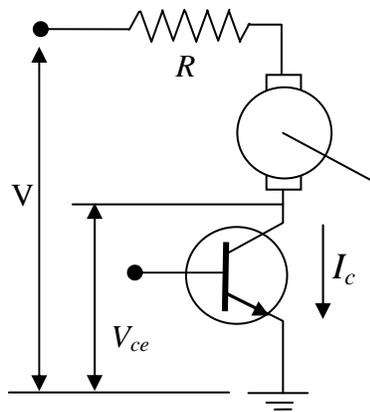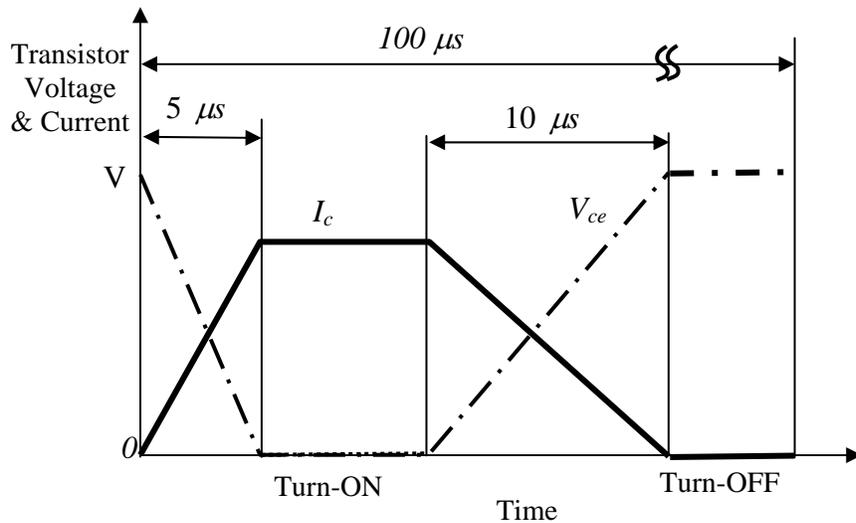

Figure 4-a DC motor connected
to a switching transistor

Figure 4-b Switching characteristics of the transistor