1. The velocity vector field \( \vec{V}(x, y, z, t) = (u, v, w) \) is given by \( \vec{V} = 2xy\hat{i} + t^2\hat{k} \). At the point (-1,2,0) evaluate the following:

(a) \((\vec{\nabla} \times \vec{V}) \times \vec{V}\)

(b) \((\vec{V} \cdot \vec{\nabla})\vec{V}\)

2. Stokes’ Theorem states that for a 2-sided surface \( S \) in three dimensions having a closed curve \( C \) as its boundary,

\[
\int_S \vec{n} \cdot (\vec{\nabla} \times \vec{V}) dS = \oint_C \vec{V} \cdot d\vec{l}.
\]

where \( \vec{V} \) is a continuously differentiable vector function, \( \vec{n} \) is the unit normal to the chosen positive side of \( S \), and \( d\vec{l} \) is in the corresponding positive direction along \( C \).

Verify the theorem using direct integration for the case where \( \vec{V} = y^2\hat{i} + x^2\hat{j} \) and \( S \) is a circular disk of radius \( R \) in the \( x-y \) plane having \( \vec{n} = \hat{k} \).

3. Suppose a fluid is rotating about an axis fixed in space at a constant angular velocity \( \vec{\omega} \). Choose the origin of a rectangular coordinate system to be on this axis. The position vector to any point in the fluid is \( \vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \). Knowing that the velocity of any point is then \( \vec{V} = \vec{\omega} \times \vec{r} \), determine the following:

(a) \( \vec{\nabla} \cdot \vec{V} \) and (b) \( \vec{\nabla} \times \vec{V} \).  

Hint: Consider \( \vec{\nabla} \cdot \vec{r} \), \( \vec{\nabla} \times \vec{r} \) and \( \vec{u} \cdot \vec{\nabla} \) (\( \vec{u} \) any vector)

4. Using the divergence theorem, evaluate \( \iiint_S \vec{F} \cdot \vec{n} dA \) over the closed surface of the cube bounded by the planes \( x = \pm 1, \ y = \pm 1, \ z = \pm 1 \) where \( \vec{F} = x\hat{i} + y\hat{j} + z\hat{k} \).