**SEAKEEPING IN RANDOM WAVES**

- Assume known the ambient wave spectral density $S_5(\omega_0)$ assumed unidirectional for simplicity.

- $\frac{1}{2}A_i^2 = S(\omega_i)\Delta \omega$

- $\int_0^\infty S_5(\omega) \, d\omega = 6_5^2 \equiv \text{Variance of the wave elevation of ambient random seastate, assumed Gaussian with zero mean.}$

- Assuming that the $\text{RAO}(\omega)$ of a seakeeping quantity $X(t)$ has been determined from a frequency domain analysis:

  $\mathbb{E}[S(t)]$ \quad \text{Linear System} \quad \mathbb{E}[X(t)]$

  Gaussian $\bar{S} = 0$, $\sigma_S^2$

  $\bar{X} = 0$, $\sigma_X^2 = \int_0^\infty S_5(\omega) |\text{RAO}|^2 \, d\omega$

  **Wienner-Khinchine**
SPECTRAL ANALYSIS WITH FORWARD-SPEED

\[ \omega = \left| \omega_0 - U \frac{\omega_0^2}{g} \cos \beta \right| \]

- Ambient wave spectral density \( S_\omega(\omega_0) \) is defined relative to the absolute wave frequency \( \omega_0 \).
- The RAO(p) is usually defined relative to the encounter frequency \( \omega \).
- The relation of \( \omega \rightarrow \omega_0 \) is not single valued. The question thus arises of what is the \( G_x^2 \) value?

Answer

→ Given \( \omega_0 \), a single value of \( \omega \) always follows.
→ The opposite is not always true. Given \( \omega \) there may exist multiple \( \omega_0 \)'s satisfying the encounter frequency relation.
→ Therefore it is much simpler to parameterize with respect to \( \omega_0 \), even when the RAO(\( \omega \)) is evaluated as a function of \( \omega \).

Proceed as follows:
INTRODUCE:

\[
\begin{align*}
\mathcal{S}_3(\omega_0) & \quad \quad \omega_0 \\
\mathcal{F}\frac{A}{A}(\omega) & \quad \quad \omega \\
\omega & = \left| \frac{\omega_0 - U \frac{\omega_0^2}{g} \cos \beta}{\omega_0} \right|
\end{align*}
\]

CONSIDER THE HEAVE MOTION OF A SHIP WITH FORWARD SPEED

SIMPLY REDEFINE THE RAO \( \omega \) AS FOLLOWS

\[
| \text{RAO}_3(\omega) | = | \text{RAO}_3 | (\omega_0 - U \frac{\omega_0^2}{g} \cos \beta) \]

\[= | \text{RAO}_3 | (\omega_0) \] , NEW FUNCTION OF \( \omega_0 \) BY VIRTUE OF THE \( \omega \leftrightarrow \omega_0 \) RELATION

THE STANDARD DEVIATION OF HEAVE FOLLOWS BY SIMPLE INTEGRATION OVER \( \omega_0 \):

\[
\sigma_3^2 = \int_0^\infty d\omega_0 \mathcal{S}_3(\omega_0) | \text{RAO}_3^*(\omega_0) |^2
\]

THE OPPOSITE CHOICE OF PARAMETRIZING THE ABOVE INTEGRAL WRT \( \omega \) ENDS UP WITH A LOT OF UNNECESSARY ALGEBRA.