The sketch shows a circular bearing pad which rests on a flat base through the intermediary of a film of viscous liquid of instantaneous thickness $h(t)$. The load $W$ causes the pad to sink slowly at the speed $S$, and this squeezes the liquid out from under the pad. Assume that $h \ll D$, that the viscosity is very high, and that the speed $S$ is very small.

- (a) Making approximations (state them precisely) consistent with these assumptions, show that the settling speed is

$$S = \frac{32 W h^3}{3 \pi \mu D^4}. \quad (6.21a)$$

- (b) An apparatus with two very flat plates of 0.3 m diameter carries a load of 100 kg on a film 0.003 cm thick. If the liquid is a heavy oil with a kinematic viscosity of $10 \ \text{mm}^2/\text{s}$ and a density of $0.93 \ \text{g/cm}^3$, estimate the speed $S$.

- (c) If the load $W$ is constant, and the gap width is $h_0$ at time zero, show that the width $h$ varies with time accordingly to

$$\frac{h}{h_0} = \left[1 + \frac{64 W h_0^2}{3 \pi \mu D^4} t\right]^{-\frac{1}{2}}. \quad (6.21b)$$

- (d) Calculate, for the initial conditions of part (b), the time (in hours) required for the gap width to be decreased to half its initial value.

- (e) Suppose now that the initial thickness is $h_0$, and that a constant upward force $F$ pulls the disk away from the base. Show that the disk will be pulled away in a time

$$t_\infty = \frac{3 \pi \mu D^4}{64 h_0^2 F}. \quad (6.21c)$$

NB When $h_0$ is very small, the time $t_\infty$ is very large. This is the basis for the phenomenon of viscous adhesion, e.g., adhesives such as Scotch tape, or the apparent adhesion of accurately-ground metal surfaces.