Problem 1.00
This problem is from “2.25 Advanced Fluid Mechanics” by Ain Sonin

Rate of change of properties measured by a probe moving through the earth’s atmosphere — plus some things about the earth and its atmosphere.

The pressure distribution in a static, constant-temperature planetary atmosphere modeled as an ideal gas is given by

\[ p = p_0 e^{-z/H} \]  \hspace{1cm} (1.00a)

where \( z \) is the altitude above a reference altitude \( z = 0 \), \( p_0 \) is the absolute pressure at \( z = 0 \), and

\[ H = \frac{RT}{Mg} \]  \hspace{1cm} (1.00b)

is a length scale that characterizes the atmosphere. Its value is determined by the strength of the gravitational acceleration and the parameters that appear in the ideal-gas equation of state,

\[ p = \rho \frac{RT}{M} \]  \hspace{1cm} (1.00c)

\( R = 8.32 \text{ JK}^{-1} \text{ mol}^{-1} \) is the universal gas constant, \( T \) is the absolute temperature (taken as constant in this model of the atmosphere), \( M \) is the molar mass of the gas (0.029 kg/mol if the gas is air), and \( g \) is the acceleration of gravity at or near the surface of the planet. For the “Standard” isothermal model of the earth’s atmosphere, \( T = 288 \text{ K} \), \( p_0 = 1.02 \times 10^5 \text{ N/m}^2 \) if \( z = 0 \) at sea level, and consequently \( H = 8.43 \text{ km} \). Note that the distribution given above is based on the assumption that \( H \ll a \), where \( a \) is the planet’s radius.

Suppose a sounding rocket or balloon equipped with a static-pressure sensor is traveling through the atmosphere with given velocity \((v_z, v_y, v_z)\).

1. In terms of the given quantities and \( z \), derive an expression for the rate of change of pressure recorded by the rocket’s sensor.
2. Evaluate this time of change at an altitude \( z = 20,000 \text{ m} \) for a rocket traveling upward through the earth’s atmosphere with a direction of 30° from the vertical and a speed of 465 m/s. (Answer: –0.273 bar/min.)
3. Suppose a rocket carries instruments that measure both the instantaneous atmospheric pressure \( p \) and the rate of change of that pressure, \( dp/dt \). Given the value of these two quantities at a particular time and \( p_0 \) and \( H \), derive expressions for the rocket’s instantaneous altitude and vertical (upward) velocity.

Additional things to think about, if you are so inclined:

4. Suppose the Earth’s atmosphere is isothermal and radially symmetric around a perfectly spherical earth with radius \( a = 6400 \text{ km} \). What is the total mass of the Earth’s atmosphere? What fraction is this of the solid and liquid parts of the planet’s mass? (Answers: \( 5.35 \times 10^{18} \text{ kg} \) and \( 8.96 \times 10^{-7} \).)
5. Show that 99% of the Earth’s atmosphere’s mass resides below an altitude of 39 km.

6. If the atmosphere heats up by 10°C, by how much will the absolute pressure at sea level change? (Answer: it will not change at all.)