2.72 Elements of Mechanical Design
Spring 2009

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Elements of Mechanical Design
Lecture 02: Review
Intent

High-level review of undergrad material as applied to engineering decision making

NOT an ME “redo” or a “how to” recitation
Main goal of 2.72 is to teach you how to integrate past knowledge to engineer a system.

Given this, how do I engineer a mechanical system?

modular → simple → complex → system

2.001, 2.002 2.003, 2.004 2.005, 2.006 2.007, 2.008
Use of core ME principles that you know...

2.001, 2.002

2.003, 2.004

2.005, 2.006

2.007, 2.008

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Impact

Understand why & how we will use parts of ME core knowledge

Problem set → Engineering
Future help

We can’t use lecture time to redo the early curriculum

BUT

We are HAPPY to help outside of lecture IF you’ve tried
Schedule and reading assignment

Reading quiz

Changing from sponge to active mode

Lecture
Mechanics
Dynamics
Heat transfer
Matrix math

Hands-on
Mechanics
Dynamics
Heat transfer
Matrix math

Reading assignment

- Shigley/Mischke
  - Sections 4.1–4.5: 08ish pages & Sections 5.1–5.5: 11ish pages
  - Pay special attention to examples 4.1, 4.4, 5.3, and 5.4
Mechanics
Free body diagrams

Useful for:
- Equilibrium
- Stress, deflection, vibration, etc...

\[ \sum \vec{F} = 0 = ma \]
\[ \sum \vec{M} = 0 = I \ddot{a} \]

Why do we ALWAYS use free body diagrams?
- Communication
- Thought process
- Documentation

How will we use free body diagrams?
- We are dealing with complex systems
- We will break problem into modules
- We will model, simulate and analyze mechanical behavior
- Integrate individual contributions to ascertain system behavior
Free body diagrams: Bearings/rails
Static: Head stock deformation
Static: Rail deformation

Model name: Lathe_structure_dynamics_example
Study name: Static
Plot type: Deformed shape Plot1
Deformation scale: 600

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**Example 1: $0 < x < a$**

**Cantilever**
- Forces, moments, & torques

**Why do we care?: Stress**
- Shear & normal
- Static failure
- Fatigue failure

**Why do we care?: Stiffness**
- Displacement
- Rotation
- Vibration $\rightarrow (k/m)^{1/2}$

**But, ends aren’t all that matters**
Example 1: $0 < x < a$

But, ends aren’t all that matters

Relating $V(x)$ & $M(x)$

- $V(x) = F$
- $M(x) = F \cdot (x - a)$
- $V(x) = \frac{d}{dx} M(x)$

Shear moment diagrams

- Solve statics equation
- Put point of import on plots
- Use $V = dM/dx$ to generate $M$ plot
- Master before spindle materials
Example 1: $0 < x < a$

Stress

\[ M(x) \overset{\downarrow}{\leftrightarrow} M(x+dx) \]

\[ \sigma_{\text{max}} \]

\[ I(x) = \frac{1}{12} b(x) \, [h(x)]^3 \]

\[ \sigma(y) = M \frac{y}{I(x)} \rightarrow \sigma_{\text{max}} = M \frac{c}{I(x)} \]

\[ |\sigma_{\text{max}}| = F \cdot (a - x) \frac{h(x)}{2} \frac{12}{b(x) \cdot [h(x)]^3} \]

\[ |\sigma_{\text{max}}| = 6 \frac{F \cdot (a - x)}{bh^2} \]

\[ \frac{6F}{bh^2} \]

\[ \frac{-6F}{bh^2} \]

\[ \sigma = 6 \frac{F \cdot (a - x)}{bh^2} \]

\[ \sigma = -6 \frac{F \cdot (a - x)}{bh^2} \]

\[ M(x) \]

\[ M(x) = -V(x) \cdot (x - a) = -F \cdot (a - x) \]

\[ \sum F = 0 = -F + V(0) \rightarrow V(0) = V(x) = F \]

\[ \sum M = 0 = +V(x) \cdot (a - x) - M(x) \rightarrow M(x) = -V(x) \cdot (a - x) \]
Group work: Generate strategy for this...
Dynamics
Vibration

Vibration principles
- Exchange potential-kinetic energy
- 2nd order system model

Blocks and squiggles…
- What do they really mean?
- Why are they important?
- How will we apply this?

Multi-degree-of-freedom system
- Mode shape
- Resonant frequency

Estimate $\omega_n$ (watch units) for:
- A car suspension system

$$\omega_n = \sqrt{\frac{k}{m}}$$
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http://www.hpiracing.com/graphics/kits/547/_MG_1962e.jpg
http://www.societyofrobots.com/images/mechanics_suspension_honda.gif
http://www.bose.com/images/learning/lc_susp_frontmodule.jpg
Vibration: MEMS device behavior

Without input shaping

With input shaping

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Vibrations: Meso-scale device behavior


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Vibrations: Reducing amplitude…

How to change $m$, $k$, and $c$?
So... where find in reality... in lathe...
Vibration: Lathe structure – 1\textsuperscript{st} mode

Model name: Lathe\_structure\_dynamics\_example
Study name: Study 1
Plot type: Frequency Plot1
Mode Shape: 1 Value = 401.72 Hz
Deformation scale: 0.005

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Vibration: Lathe structure – 2nd mode

Model name: Lathe_structure_dynamics_example
Study name: Study 1
Plot type: Frequency Plot1
Mode Shape: 2 Value = 761.55 Hz
Deformation scale: 0.005
Heat transfer
Thermal growth errors

For uniform temperature

\[ \Delta L = \alpha L_o \Delta T \]

**STEEL : 12L14** → \( \Delta L = 11.5 \times 10^{-6} \frac{m}{m^\circ C} L_o \Delta T \)

**ALUMINUM : 6061 T6** → \( \Delta L = 23.6 \times 10^{-6} \frac{m}{m^\circ C} L_o \Delta T \)

**POLYMER : Delrin** → \( \Delta L = 100 \times 10^{-6} \frac{m}{m^\circ C} L_o \Delta T \)
Convection and conduction

Convection:

\[ \dot{q} = h A_{surface} (T - T_\infty) \]

Why do we care?

- Heat removal from cutting zone
- Heat generation in bearings
- Thermal growth errors

Conduction:

\[ \dot{q} = k A_{cross} \frac{dT}{dx} \]

Why do we care?

- Heat removal from cutting zone
- Heat generation in bearings
- Thermal growth errors

Common k values to remember

- Air \hspace{1cm} 0.026 \text{ W/(m}^\circ\text{C)}
- 12L14 \hspace{1cm} 51.9 \text{ W/(m}^\circ\text{C)}
- 6061 T6 \hspace{1cm} 167 \text{ W/(m}^\circ\text{C)}
Thermal resistance

Thermal resistance

\[ \dot{q} = \frac{\Delta T}{R_T} \]

- Convection

\[ \dot{q} = \frac{(T - T_\infty)}{(h A_{\text{surface}})^{-1}} \quad \rightarrow \quad R_T = \frac{1}{h A_{\text{surface}}} \]

- Conduction

\[ \dot{q} = dT \frac{k A_{\text{cross}}}{dx} \quad \rightarrow \quad R_T = \frac{dx}{k A_{\text{cross}}} \]
Biot (Bi) number

Ratio of convective to conductive heat transfer

\[
\frac{\dot{q}_{\text{convection}}}{\dot{q}_{\text{conduction}}} = \frac{(h A \Delta T)_{\text{convection}}}{(k A \frac{\Delta T}{L})_{\text{conduction}}} \quad \Rightarrow \quad Bi = \frac{hLc}{k}
\]

Why do we care?

Low Bi

High Bi
Example of thermal errors

For:

- \( h = 0.1 \text{ W}/(\text{m}^2\text{oC}) \)
- Bearing \( T = 150 \text{ oF} \)
- Chip \( T = 180 \text{ oF} \)

For:

- \( h = 50 \text{ W}/(\text{m}^2\text{oC}) \)
- Bearing \( T = 150 \text{ oF} \)
- Chip \( T = 180 \text{ oF} \)
Example of thermal errors

For:

- $h = 0.1 \text{ W/(m}^2\text{C)}$
- Bearing $T = 150 ^\circ \text{F}$
- Chip $T = 180 ^\circ \text{F}$

For:

- $h = 5000 \text{ W/(m}^2\text{C)}$
- Bearing $T = 150 ^\circ \text{F}$
- Chip $T = 180 ^\circ \text{F}$
Types of errors
Machine system perspective

System-level approach
Linking inputs and outputs
Measurement quality

\[
\begin{bmatrix}
C_1 & C_2 & C_3 \\
C_4 & C_5 & C_6 \\
C_7 & C_8 & C_9
\end{bmatrix}
\]

- Desired outputs
  - Motion, location, rotation
  - Cutting forces
  - Speeds

- Measured outputs
  - Motion, location, rotation
  - Cutting forces
  - Speeds

- Actual outputs
  - Motion, location, rotation
  - Cutting forces
  - Speeds

Perceived Error
Real error

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Errors....

Accuracy
- The ability to tell the “truth”

Repeatability
- Ability to do the same thing over & over

Both
- 1st make repeatable, then make accurate
- Calibrate

Determinism
- Machines obey physics!
- Model \( \rightarrow \) understand relationships

\[
\begin{bmatrix}
C_1 & C_2 & C_3 \\
C_4 & C_5 & C_6 \\
C_7 & C_8 & C_9
\end{bmatrix}
\]

\[
[Outputs] = \begin{bmatrix}
C_1 & C_2 & C_3 \\
C_4 & C_5 & C_6 \\
C_7 & C_8 & C_9
\end{bmatrix} [Inputs]
\]

- Understand sensitivity

\[
[\Delta Outputs] = J [\Delta Inputs]
\]

Range
- Furthest extents of motion

Resolution
- Smallest, reliable position change

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Categorizing error types

Systematic errors
- Inherent to the system, repeatable and may be calibrated out.

Non-systematic errors
- Errors that are perceived and/or modeled to have a statistical nature
- Machines are not “random,” there is no such thing as a random error

Consider the error for each set below
- Link behavior with systematic and non-systematic errors.
Exercise
Exercise

Due Tuesday, start of class:

Lathe components

- Rough sketch(es) of lathe
- Annotate main components

1 page bullet point summary of where need to use:

- 2.001, 2.002  2.003, 2.004  2.005, 2.006  2.007, 2.008

Rules:

- You may not re-use examples from lecture!
- You are encouraged to ask any question!
- You may work in groups, but must submit your own work
Group work: Generate strategy for this...