In tonal music, usually think of intervals—that is the distance in pitch space between two notes—as a two-part measure, first the number of lines or spaces separating the two pitches (unison if on the same line, second if one[], third if two, etc.) and then based on the quality of sound (major, minor, diminished, augmented). We call an interval between two notes major if it appears in a major scale beginning on the lower of the two notes. We call an interval between two notes minor if it appears in a major scale beginning on the higher of the two notes. For instance, we call the first of the following two intervals a major sixth because A appears in a C-major scale, but not vice-versa. However, we call the second interval a minor sixth because C appears in an A♭-major scale:

The intervals of the fourth, fifth, and eighth (octave) are exceptions. A fourth or fifth which appears in both a major and minor scale, such as C-G or A♭-E♭, is called “perfect”. An interval smaller by one half-step than a perfect (or minor) interval we call diminished, while an interval larger than a perfect (or major) interval we call augmented, as follows:

In traditional harmony, all augmented and diminished intervals are considered dissonant, while the perfect fifth and the perfect octave are consonant (the perfect fourth is generally considered a dissonance although it retains some consonant qualities). The other intervals are consonant or dissonant depending on their scalar distance and not according to their quality (i.e., thirds and sixths are consonances whether major or minor. Seconds and sevenths are dissonances).

These definitions of consonance and dissonance, while somewhat rooted in acoustical phenomenon¹ are generally culturally defined: in other parts of the world, sixths are not considered consonances while seconds are, for instance, or other notes not available in our system (half-flats and so on) are might be considered consonances.

The nomenclature we have used (“Major second” etc.) ties notes to dissonant or consonant qualities and to scales. Thus it should not be surprising that the atonal composers, and those who analyze atonal works, found these labels misleading or inadequate to the study of their works.

¹ There is no known human culture which does not have the octave as an interval which is in some ways consonant.
They chose a definition of interval entirely rooted in the number of half-steps or semitones between two notes, as follows:

(T = Ten, E = Eleven). Since composers were often not interested in octave displacement (that is to say, one octave was as good as any other; we call this “octave equivalence”), the list of intervals often is said to cycle over again after 11, like the hours of a clock (except we’ve put 0 instead of 12). When we talk about notes without reference to their octave, we call this the “pitch class” of a note. For instance, all “C’s” on the piano are part of the pitch class “C”. Note also how the distance from C-D♭ is 1 and the distance from C-C♯ is also 1; that is to say, how we spell the note does not matter when we measure intervals in semitones.

Another useful application of semitone intervals is that we can label all notes by their interval to a reference note (which is almost always C), so, for instance we can label the pitch class E♭ as 3 and A as 9. The note D♯ would also be 3 and B♭ would be 9. This useful shorthand allows chords to be expressed compactly. For instance the C-major triad (C-E-G) can be called (047) — verify this labeling from the chart above. The D♭-minor triad (D♭-F♭-A♭) could be called (36T). These labels might not seem much of a shorthand so far, since we already have good names for these chords (C-major, etc.) The usefulness comes in describing arbitrary collections. For instance, the six-note collection found in Schoenberg opus 19, no. 6:

Could be called “G-C-F-A-F♯-B” or (70596E), which is shorter. Another thing we can do with such a collection is rearrange it so the components are in ascending order (05679E). This lets us see that we have a number of notes bunched up (5679 or F-F♯-G-A) with two notes further out. We can then look at other collections in the same piece, such as that found at the fermata of measure 6:

And see that this one is (42605T) or written in ascending order (02456T). Again we can see that we have a number of notes bunched together with a few outliers, so these two chords, both found in prominent positions in the piece are in a sense related. (There are many more sophisticated ways of comparing these two collections, such as finding normal and prime forms, looking at interval content and interval vectors, subcollections, etc. If you’re interested in these types of things, a twentieth-century analysis course is probably just the trick. We’re going to be moving a bit too fast in this class to get into such things here).
Interval number sets also allow us to see if two sets of pitches are related to each other by transposition. That is to say, is there an interval up or down we can move a chord or melody or pitch set to match another chord, melody, or pitch set. Consider these two melodies from Webern, *Fünf Sätze*, movement four:

![Melody Example](image)

We can write the first melody’s pitch set as (046E17T). The second melody (remember the alto clef!) is F-A-B-E-F#-C-E♭, or (59E4603). What we can ask is can we add some number to every member of the first melody to get the second (or vice versa). The answer is yes! The pitches of the first melody transform into those of the second if we add 5 to each of them:

\[
\begin{align*}
0 & \quad 4 & \quad 6 & \quad E & \quad 17 & \quad T \\
+ & \quad 5 & \quad 5 & \quad 5 & \quad 5 & \quad 5 & \quad 5 \\
\hline
5 & \quad 9 & \quad E & \quad 4 & \quad 6 & \quad 0 & \quad 3
\end{align*}
\]

(recall that E = 11, so E+5 = 16, and 16:00 is the same as 4:00)

which is the second melody! We sometimes say that first melody’s pitches transform into the seconds under the T₅ operation, meaning transpose up 5 half-steps (and down an octave!). The second melody is related to the first under T₇ operation (double check this for yourself).

A big difference between an analysis class and a musicology class is that we now need to ask further questions. Why does Webern choose to use two transpositionally related melodies in this piece? What is the effect of these relations on the listener? Why does he not choose to use the same rhythms between the two statements? Are there historical precedents for this type of operation? Can we perceive the relationship between these two statements? (I think we can in this case, but we will ask soon, “If we can’t, does it matter?”)
**The Worksheet proper**

Please detach this sheet and bring it, completed, to class on Tuesday (9/26).

1. Give the pitch-class number for each of the following pitches:

   \[
   \begin{align*}
   &\text{\#-5} \\
   &\text{-3} \\
   &\text{-1} \\
   &\text{0} \\
   &\text{1} \\
   &\text{2} \\
   &\text{3} \\
   &\text{4} \\
   &\text{5} \\
   &\text{6} \\
   &\text{7} \\
   &\text{8} \\
   &\text{9} \\
   &\text{10} \\
   &\text{11} \\
   
   \end{align*}
   \]

2. Notate the following pitch-class numbers on the staves below in two different ways, employing different clefs, octaves, and accidentals: (a) 0 E 7 9 2 T 8 1 4 6 6 9 8

   \[
   \begin{align*}
   &\text{\#-5} \\
   &\text{-3} \\
   &\text{-1} \\
   &\text{0} \\
   &\text{1} \\
   &\text{2} \\
   &\text{3} \\
   &\text{4} \\
   &\text{5} \\
   &\text{6} \\
   &\text{7} \\
   &\text{8} \\
   &\text{9} \\
   &\text{10} \\
   &\text{11} \\
   
   \end{align*}
   \]

   (b) 2 1 9 T 5 3 4 0 8 7 6 E

   \[
   \begin{align*}
   &\text{\#-5} \\
   &\text{-3} \\
   &\text{-1} \\
   &\text{0} \\
   &\text{1} \\
   &\text{2} \\
   &\text{3} \\
   &\text{4} \\
   &\text{5} \\
   &\text{6} \\
   &\text{7} \\
   &\text{8} \\
   &\text{9} \\
   &\text{10} \\
   &\text{11} \\
   
   \end{align*}
   \]

3. Give the interval (0-11) between each successive pitch of exercise 2(a)(b). (e.g., 0 to E = E, E to 7 = 8)

4. Write the pitch set (e.g., (49E20)) found at the end of the third system of Schoenberg, op. 19, no. 2 from lowest on the page to highest:

   now write it in ascending order (e.g., (0249E):)
5. Find a third melody in *Fünf Sätze* no. 4 which is transpositionally related to the two given on page 3 of this handout. Notate it on the staff below, label the pitch class numbers, and show (as above) how its pitches can be transformed into those of the melody of m. 6:

______________________________
______________________________
______________________________

6. Create a melody (i.e., not an ascending set of pitches) using the pitch classes which are NOT present in the melody of m. 6 of *Fünf Sätze*.

______________________________
______________________________
______________________________

Write the pitch set of your melody in ascending order:
(We call the pitches not in a melody the “complement set” of that melody)

What intervals seem most important in the complement set of the melody?

7. Write in ascending order the pitch set of a C-major scale:

Give the complement set of the C-major scale:

What name do we give to this scale?² (Google “Five-note scales” if you’re stuck.)

² Optional! Circle the correct answer to the statement, “How cool is it that these two great scales are complement relations?” a. Extremely! b. Wicked! c. Unearthly d. All of the Above.