Problem Set 5
Intro to Sustainable Energy 2.650/10.291/22.081
&
Sustainable Energy 1.818/2.650/10.391/11.371/22.811/ESD.166

For each of the problems you work out provide a list of sources for any data you used, as well as your assumptions. Be sure to mark which course number you are registered for on your solution. You can turn in the homework online (via Stellar) or in class.

Intro to SE Students: Pick any 2 of the 4 problems to solve.
SE Students: Pick 3 of the 4 problems to solve.

1. Consider two power plants, each generating electric power $\dot{W}_e$ at 300 MW, and operating at an annual capacity factor $CF$ of 80%. One plant is fueled by bituminous coal and has an overall efficiency $\eta$ of 35%. The other plant is combined-cycle, gas-fired system for which the overall efficiency is 55%. Assuming higher heating values (HHV, kJ/kg-fuel) of 28,000 and 54,000 and carbon contents (CC, Mt-C/Mt-fuel) of 0.67 and 0.74 for the coal and natural gas, respectively, determine the annual fuel requirement and CO$_2$ emissions for each plant. Express your results in tonnes (Mt). If coal and natural gas are priced at $80/Mt and $10/MMBtu, what is the annual fuel cost $FC$ for each system? If CO$_2$ emissions from the gas-fired system were used as the standard for capping power plant emissions, by how much would the standard be exceeded for the coal-fired plant? Express your result in Mt-CO$_2$. To discourage business-as-usual practices in the design and operation of coal-fired plants, what carbon taxes would you recommend for emissions exceeding the prescribed standard. Express your result in $ per Mt-C.

2. Consider that breeder reactors are to be built using $^{233}$U (obtained from $^{232}$Th) and $^{239}$Pu, respectively.
   a. What is the maximum conversion ratio that could be achieved in principle with each fuel? (Assume that the reactor consists of pure fuel and that no neutrons escape from it.)
   b. What factors cause real-world conversion ratios to be lower than the values you calculated in part a?
   c. Explain why “reactor grade” plutonium is not desired for use in a weapon.

3. In class we have noted that public participation in nuclear power plant licensing processes has led to costly delays, and that such participation is justified by some, partially upon the basis that alternative fora for de-facto determination of the effective national energy policies are largely unavailable within the U.S. political system. If public participation in licensing were to be eliminated, it might be wise to create such fora. Propose two such methods for determination of national energy policies and provide a critique contrasting them. (Hint, you might examine how other countries do this.)

4. Consider a gas-fired power plant for which an air-standard Brayton cycle can be assumed. Ambient air enters the compressor at $T_1 = 300$ K and $p_1 = 100$ kPa, experiences a compression ratio of $p_2/p_1 = 15$, and is heated to a turbine inlet temperature of $T_3 = 1600$ K. The compressor and turbine each have an isentropic efficiency of 80%.
a. Per unit mass of air, determine the work input and output for the compressor and turbine, respectively, the heat addition, and the thermal efficiency of the cycle.
b. For a power plant generating 200 MW, with a turbine generator efficiency of \( \eta_{\text{gen}} = 0.95 \), what is the required air mass flow rate \( \dot{m}_a \) in kg/s?

c. Using a lower heating value of 45,000 kJ/kg for natural gas, determine the mass flow rate \( \dot{m}_g \) in kg/s at which gas must be supplied to the combustor.

d. If the gas has a carbon content of 0.74 kg-C/kg-fuel, what is the corresponding rate of CO₂ production?

Suggestion: Isentropic exit conditions for the compressor and turbine can be determined from tabulation of the ideal gas properties of air and use of the property \( p_r(T) \). Specifically,

\[
p_{r2} = \frac{p_2}{p_1} \quad p_{r3} = \frac{p_3}{p_1} \quad p_{r1} = \frac{p_1}{p_3}
\]

Alternatively, if a constant specific heat is assumed, in addition to the assumption of ideal gas behavior, exit conditions can be determined from the relations

\[
\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{0.286} \quad \frac{T_4}{T_3} = \left( \frac{p_4}{p_3} \right)^{0.286}
\]