Problem 1 (30%) – Hydraulic analysis of the PWR primary system at cold zero-power conditions

A greatly-simplified schematic of the PWR primary system is shown in Figure 1. The core and steam generators are represented by two form losses of coefficients 7 and 4, respectively. The loop can be modeled as a series of four identical round tubes of 1.45 m ID and 10 m length. The flow within the loop is driven by a pump that delivers a constant head, $\Delta P_{\text{pump}} = 200 \text{ kPa}$, regardless of the flow.

![Figure 1. Simplified schematic of the PWR primary system](image)

You are to evaluate the hydraulic behavior of the system at cold zero-power conditions. In this situation the fluid can be considered isothermal at 20°C and atmospheric pressure. The properties of water at this temperature and pressure are reported in Table 1.

i) Calculate the steady-state mass flow rate in the system. Clearly state all your assumptions. (10%)

ii) Now consider flow start-up from stagnant conditions. At $t=0$ the pump is turned on and the flow is established. Calculate the time it takes for the mass flow rate to reach 50% of its steady-state value. (15%) *(Hint: use the following integral)*

$$\int \frac{dx}{c^2 - x^2} = \ln \left( \frac{c + x}{c - x} \right)^{\frac{1}{2c}}$$

iii) A nuclear engineer wishes to simulate the PWR primary system by means of an experimental flow loop with the same form coefficients and geometrically similar, but of
1/10 scale (the pump head is also scaled down to 1/10). Would such loop have the same
time constant of the PWR primary system? (5%)

Assumptions:
- Neglect the acceleration and friction terms ($F_{\text{acc}}$ and $F_{\text{fric}}$, respectively) in the momentum
equation.

Table 1. Water properties at $20^\circ$C.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_l$</td>
<td>1,000 kg/m$^3$</td>
</tr>
<tr>
<td>$C_{p_l}$</td>
<td>4.2 kJ/(kg⋅K)</td>
</tr>
<tr>
<td>$k_l$</td>
<td>0.6 W/(m⋅K)</td>
</tr>
<tr>
<td>$\mu_l$</td>
<td>1.0×10$^{-3}$ Pa⋅s</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.2×10$^{-4}$ 1/K</td>
</tr>
</tbody>
</table>
Problem 2 (25%) – Surface tension effects in borated water draining from a BWR Standby Liquid Control Tank.

BWRs have a Standby Liquid Control Tank (SLCT) containing highly-borated water at room temperature that can be injected into the core, should the control rods fail to shutdown the reactor during an accident. Over a long period of time, borated water corrosion has created a small round hole of 0.5 mm diameter on the bottom of the SLCT (Figure 2a). The contact angle between borated water and the SLCT material is $\theta = 120^\circ$. The surface tension of borated water at room temperature is 0.07 N/m, and its density is about 1,000 kg/m$^3$. The initial liquid level in the SLCT is 1 m.

![Diagram of SLCT with borated water, hole, and liquid level](image1)

Figure 2. The SLCT.

i) Assuming that the SLCT top is open to the atmosphere, would you expect the borated water to completely drain from the hole? (10%)
   - If so, explain why.
   - If not, calculate the level at which draining would stop.

ii) Now assume that the contact angle is $60^\circ$. Does the tank drain completely? Explain. (5%)

iii) To prevent draining, a fellow MIT nuclear engineering student suggests sealing the tank top and put a cover gas (Figure 2b). Would this in fact prevent draining? Does the contact angle affect your answer? (10%)
Problem 3 (25%) – Flow split between a heated and an adiabatic channel.

Consider the two parallel channels shown in Figure 3. They are connected only at the inlet and outlet plena, and both have flow area $A$, equivalent diameter $D_e$ and length $L$. Channel 1 is heated ($\dot{Q}$ is the total heat rate), while channel 2 is adiabatic. Channel 1 has an orifice at the inlet (of form loss coefficient $K$). The boundary conditions are as follows:

- The inlet plenum temperature is $T_o$
- The total mass flow rate is $\dot{m}_{tot}$
- The outlet plenum pressure is $P_L$

The fluid specific heat and thermal expansion coefficient are $c_p$ and $\beta$, respectively. The density of the fluid can be calculated by means of the Boussinesq approximation with $T_o$ and $\rho_o$ as the reference temperature and density, respectively.

![Figure 3. Parallel channels connected at plena.](image)

i) Find an expression for the mass flow rate in channel 1 in terms of the heat rate, geometry and properties only. (15%) *(Hint: assume steady-state upflow in both channels)*

ii) Find an expression for $\dot{Q}$ at which the mass flow rate in channel 2 becomes zero. (5%)

iii) What happens to the flow in channel 2, if the heat rate in channel 1 is increased beyond the threshold calculated in “ii”? (5%) *(Note: provide only a qualitative answer)*

**Assumptions:**
- Heating in channel 1 is axially uniform.
- Assume single-phase flow in the system.
- Neglect acceleration and friction terms in both channels.
- All thermophysical properties (except density) can be considered independent of temperature.
Problem 4 (20%) – Quenching experiments to simulate boiling heat transfer during a LB-LOCA.

To simulate boiling heat transfer on the surface of the fuel pins during a Large-Break Loss Of Coolant Accident (LB-LOCA) in a PWR, a nuclear engineer has designed a very simple quenching experiment, in which a small copper sphere (∼1 cm diameter) is heated up to very high temperatures (∼1,000°C), and then dropped in a large pool of water at atmospheric pressure.

i) What are the differences between the experiment and the actual reactor situation that are likely to have an effect on boiling heat transfer? (5%)

ii) Write the energy conservation equation describing the temperature history (T vs. t) of the copper sphere during a quenching experiment? (5%) (Hint: neglect the temperature gradient within the sphere, describe boiling heat transfer at the surface of the sphere by means of a heat transfer coefficient, and assume that the water bulk is saturated)

iii) The boiling curve for the experimental conditions is shown in Figure 4. Provide a qualitative sketch of the sphere temperature history for an initial temperature of 1,500°C. (10%)

![Figure 4. Boiling curve for a sphere in saturated water at 1 atm.](image)