A Simple Approximation

1. Instead of choosing $F(\psi)$ so $q(\psi)$ is the same everywhere, we choose a simpler $F(\psi)$ so that only $q(0)$ and $q(a)$ remain the same (as $\beta_t$ increases).

2. Choose $dp/d\psi = \text{const}, dF^2/d\psi = \text{const}$. This is the same model we have already investigated.

3. The model has only two free parameters: $A, C \rightarrow \beta_t, q_*$. 

4. Thus, as $\beta_t$ increases, there is only one degree of freedom, $q_*$, remaining.

5. Therefore, we cannot adjust $q_*$ so that both $q_0$ and $q_a$ remain fixed: this would be an overdetermined system.

6. We make an ultra simple approximation and choose $q_*$ so that only $q_a$ remains fixed. This prevents the formation of a separatrix which requires $q_a \rightarrow \infty$.

HBT Equilibrium

\[ \mu_0 \rho = \beta_t B_0^2 \left(1 - \rho^2\right) \left[1 - \nu \rho \cos \theta\right] \]

\[ B_\theta = \frac{\varepsilon B_0}{q_*} \left[ \rho + \nu \frac{\rho^2}{2} \left(3\rho^2 - 1\right) \cos \theta \right] \]

\[ \hat{B}_\theta = \frac{\varepsilon B_0}{q_*} \left[ \frac{l}{\rho} + \nu \frac{1}{2} \left(1 + \frac{1}{\rho^2}\right) \cos \theta \right] \]

\[ q_a = \frac{q_*}{\left(1-v^2\right)^{1/2}} \]

\[ \nu = \frac{\beta_t q_*^2}{\varepsilon} \]

\[ \rho = r/a \]

1. HBT: Express all quantities in terms of $\beta_t, q_* - 1/I$

2. FCT: Express all quantities in terms of $\beta_t, q_a$ (held fixed). Examine the behavior as $\beta_t$ increases. Are there any equilibrium limits?
Procedure

1. Define \( \nu_\ast = \frac{\beta_t q_a^2}{\epsilon} \) \( \propto \beta_t \) since \( q_a \) is held fixed in the FCT.

2. \( \nu_\ast \) is the heating parameter: as \( \beta_t \) increases, \( \nu_\ast \) increases.

3. For the HBT: \( \nu = \frac{\beta_t q_a^2}{\epsilon} \) \( \propto \beta_t \) for fixed \( I \).

4. \( \nu \) is the heating parameter for fixed \( I \): as \( \beta_t \) increases, \( \nu \) increases.

Relation between \( \nu \) and \( \nu_\ast \)

1. \( \nu = \frac{\beta_t q_a^2}{\epsilon} = \frac{\beta_t q_a^2}{q_a^2} = \nu_\ast (1 - \nu^2) \)

2. \( \nu^2 + \frac{\nu}{\nu_\ast} - 1 = 0 \)

\[
\nu = \frac{2\nu_\ast}{(1 + 4\nu_\ast^2)^{1/2} + 1}
\]

Compute the Physical Quantities in Terms of \( \nu_\ast \) and Compare with the HBT

1. \( I \propto 1/q_\ast \)

   a. HBT: \( \frac{1}{q_\ast} = \) const. fixed \( I \)

   b. FCT: \( \frac{1}{q_\ast} = \frac{1}{q_a} \left( \frac{1}{1 - \nu^2} \right)^{1/2} = \frac{1}{q_a} \left( \frac{\nu_\ast}{\nu} \right)^{1/2} \)

   \[
   \frac{1}{q_\ast} = \frac{1}{q_a} \left[ \frac{1 + (1 + 4\nu_\ast^2)^{1/2}}{2} \right]^{1/2}
   \]

2. \( B_v \)

   a. HBT: \( B_v = \frac{\mu_0 I}{4\pi R_0} \beta_p = \frac{\mu_0 I}{4\pi R_0} \frac{\epsilon B_0}{q_\ast} \frac{\epsilon B_0}{2} = \frac{\epsilon B_0}{q_\ast} \nu \)

   b. FCT: \( B_v = \frac{\epsilon B_0}{2} \frac{\nu}{q_\ast} = \frac{\epsilon B_0}{2} \frac{1}{q_a} \left[ \frac{1 + (1 + 4\nu_\ast^2)^{1/2}}{2} \right]^{1/2} \frac{2\nu_\ast}{1 + (1 + 4\nu_\ast^2)^{1/2}} \)
\[ B_\nu = \frac{\varepsilon B_0}{2} \frac{v_*}{q_\beta} \left[ \frac{2}{1 + (1 + 4v_*^2)^{1/2}} \right]^{1/2} \]

3. \( \rho_\nu \)
   a. HBT: \( \rho_\nu = \frac{1}{\nu} \left[ 1 + (1 - \nu^2)^{1/2} \right] \)
   b. FCT: \( \rho_\nu = \frac{1 + (1 + 4\nu_*^2)^{1/2}}{2\nu_*} \left[ 1 + \left( \frac{2}{1 + (1 + 4\nu_*^2)^{1/2}} \right)^{1/2} \right] \)

4. Define the plasma evolution in \( \beta_t - q_* \) space as \( \beta_t \) increases
   a. HBT: \( \frac{\beta_t q_*^2}{\varepsilon} = \nu \)
      \[ q_* = \text{const.} \]
   b. FCT: \( \frac{\beta_t q_*^2}{\varepsilon} = \nu_* \)  
      \[ (1) \]
      \[ \frac{1}{q_*} = \frac{1}{q_\beta} \left[ \frac{1 + (1 + 4\nu_*^2)^{1/2}}{2} \right]^{1/2} \]  
      \[ (2) \]
   c. Solve (2) for \( \nu_* \) and substitute into (1) to give \( \beta_t = F(q_*) \)
      \[ \nu_*^2 = q_\beta^2 \left[ \frac{q_*^2}{q_\beta^2} - 1 \right] \]
      \[ \frac{\beta_t q_\beta^2}{\varepsilon} = \left[ \frac{q_*^2}{q_\beta^2} \left( \frac{q_*^2}{q_\beta^2} - 1 \right) \right]^{1/2} \]
Plot the Results

1. \( I \propto \nu \)

As \( \nu \) increases, \( I \) increases. This helps to prevent the separatrix from moving onto the plasma surface since less vertical field is required to maintain toroidal force balance.
3. $B_v$

Less vertical field is required. The separatrix stays away from the plasma surface.

4. $\rho_S$

No equilibrium limit. The separatrix does not move onto the plasma surface.
Summary

1. General HBT: covers all permissible $\beta_t/\varepsilon, q_*$ space

2. HBT at fixed $I$: exhibits an equilibrium limit

3. FCT at fixed $q_a$: no equilibrium limit