8.01L SUMMARY OF EQUATIONS

Note: Quantities shown in **bold** are vectors.

\[ \mathbf{v} = \frac{dr}{dt} \quad \mathbf{a} = \frac{d\mathbf{v}}{dt} \]

For constant acceleration \( \mathbf{a} \), if at \( t = 0 \) \( \mathbf{r} = \mathbf{r}(0) \) and \( \mathbf{v} = \mathbf{v}(0) \):

\[ \mathbf{v} = \mathbf{v}_0 + \mathbf{a}t \]
\[ \mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0t + \frac{1}{2} \mathbf{a}t^2 \]

Circular motion at constant speed \( a = v^2/r = \omega^2 r \) (Centripetal acceleration, points towards center of circle, \( \omega \) is angular speed in radians per second)

Adding relative velocities ("wrt" is short for "with respect to"): \( \mathbf{v}_{A\,\text{wrt}\,B} + \mathbf{v}_{B\,\text{wrt}\,C} = \mathbf{v}_{A\,\text{wrt}\,C} \)

\[ F! = 0 \quad a = 0 \]
\[ F = ma \quad F = \frac{dp}{dt} \] (Newton's first law)

\[ \mathbf{p} = m\mathbf{v} \] (momentum)

\[ J = \int F \, dt = \int \frac{dp}{dt} \, dt = p_2 - p_1 \] (impulse)

\[ \mathbf{r}_{cm} = \frac{\sum m_i \mathbf{r}_i}{\sum m_i} \] (position of center of mass)

\[ \mathbf{F} = -k\mathbf{x} \] (spring force) \( f \leq \mu N \) (Friction force relative to Normal force)

\[ \mathbf{F} = -\frac{GMm_i}{r^2} \hat{r} \] (gravitational force between two particles)

\[ W = \int \mathbf{F} \cdot d\mathbf{r} \] (work done by force \( \mathbf{F} \))

\[ W_{\text{other}} = \Delta E = E_f - E_i \quad E = KE + PE \] (work-energy theorem)

\[ F_x = -\frac{dU}{dx} \] (force derived from potential energy)

Potential Energies: \( U = \frac{1}{2}kx^2 \) (spring force)

\[ U = \frac{-GMm}{r} \] (gravitational, general) \( U = mgh \) (gravitational, near Earth)

\[ \omega = \sqrt{\frac{k}{m}} \quad x = A\cos(\omega t + \phi) \]

\[ v = -A\omega \sin(\omega t + \phi) \quad T = \frac{2\pi}{\omega} \] (Equations for Simple Harmonic Motion)

\[ P_2 + \rho g y_2 = P_1 + \rho g y_1 \] (Pascal’s Law: pressure versus height in a liquid with velocity = 0)

\[ P + \frac{1}{2} \rho v^2 + \rho g y = \text{constant} \] (Bernoulli’s equation)

\[ A_2 v_2 = A_1 v_1 \] (continuity equation)

\[ PV = NkT = nRT \] (ideal-gas law)

\[ \left\{ \frac{1}{2}mv^2 \right\} = \frac{1}{2} kT \] (definition of kinetic temperature)

\[ F_B = \rho_f V_f g \] (buoyancy force, \( f \)=fluid displaced)
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Translational</th>
<th>Rotational (about axis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity, acceleration</td>
<td>( \mathbf{v}, \mathbf{a} )</td>
<td>( \mathbf{\omega}, \mathbf{\alpha} ) (( \mathbf{v}=R\mathbf{\omega}, \mathbf{a}=R\mathbf{\alpha} ))</td>
</tr>
<tr>
<td>Mass</td>
<td>( M = \sum m_i )</td>
<td>( I = \sum m_i R_i^2 )</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>( \frac{1}{2} MV^2 )</td>
<td>( \frac{1}{2} I \omega^2 )</td>
</tr>
<tr>
<td>Net force</td>
<td>( \sum \mathbf{F}^\text{ext} = M \mathbf{a}_\text{cm} )</td>
<td>( \sum \mathbf{r}^\text{ext} = I \mathbf{\alpha} )</td>
</tr>
<tr>
<td>Momentum</td>
<td>( \mathbf{p} = m \mathbf{v} )</td>
<td>( \mathbf{L} = \mathbf{r} \times \mathbf{p} ) or ( \mathbf{L} = I \mathbf{\omega} )</td>
</tr>
</tbody>
</table>

\[ \mathbf{\tau} = \mathbf{r} \times \mathbf{F} = \frac{d\mathbf{L}}{dt} = I \mathbf{\alpha} \quad |\mathbf{r}| = r \sin(\phi) = F_{r_x} \quad (\text{torque equations}) \]

\[ \mathbf{L} = \mathbf{r} \times \mathbf{p} \quad |\mathbf{L}| = mr\sin(\phi) \quad (\text{angular momentum of point particle}) \]

\[ \mathbf{L} = I \mathbf{\omega} \quad (\text{angular momentum for solid object}) \]

\[ I_i = I_{c.m.} + Md_i^2 \quad (\text{parallel axis theorem}) \]

\[ I = \frac{1}{2} MR^2 \quad (\text{cylinder around center}) \quad I = \frac{2}{3} MR^2 \quad (\text{solid sphere around center}) \]

\[ I = \frac{1}{2} ML^2 \quad (\text{rod around center}) \quad I = \frac{1}{3} ML^2 \quad (\text{rod around end}) \]

\[ KE = \frac{1}{2} M \omega_{CM}^2 + \frac{1}{2} I_{CM} \omega^2 \quad (\text{kinetic energy for object moving and rolling}) \]

\[ KE = \frac{1}{2} I_{\text{pivot}} \omega^2 \quad (\text{kinetic energy for object rotating around a fixed pivot}) \]

**Physical Constants:**

\[ g = 9.8 \text{ m/s}^2 \quad \text{Use the approximate value } g = 10 \text{ m/s}^2 \text{ where told to do so.} \]

\[ G = 6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2 \]

\[ k = 1.38 \times 10^{-23} \text{ J/K} \quad R = 8.31 \text{ J/(mol. K)} \]

\[ 0^\circ C = 273^\circ K \]

**Density of water:** \( 1,000 \text{ kg/m}^3 \)

**Atmospheric pressure:** \( 1.0 \times 10^5 \text{ Pa} \)

**Conversion reminder:**

\[ \pi \text{ radians} = 180^\circ \]

**Lazy Physicist’s Favorite Angle:** (to be used when calculators are not allowed):

\[ 36.9^\circ \text{ and } 53.1^\circ \text{ are the angles of a 3-4-5 right triangle so:} \]

\[ \sin(36.9^\circ) = \cos(53.1^\circ) = 0.60 \quad \cos(36.9^\circ) = \sin(53.1^\circ) = 0.80 \]

\[ \tan(36.9^\circ) = 0.75 \quad \tan(53.1^\circ) = 1.33 \]

**Other (possibly) Useful Trig Functions:**

\[ \cos(30^\circ) = \sin(60^\circ) = \sqrt{3}/2 \quad \sin(30^\circ) = \cos(60^\circ) = 1/2 \]

\[ \cos(45^\circ) = \sin(45^\circ) = 1/\sqrt{2} \]

**Solution to a Quadratic Equation:** If \( ax^2 + bx + c = 0 \) then \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \)