Quiz #4 preparations

- Quiz 4: Wed, 5/4, 10AM,
  - 1 sheet with formulae etc
  - No books, calculators
- Evening review: Tue, 5/3, 7PM
- Tutoring:
  - Angel Solis, Mon + Tue, 5/2, 5-7PM,

Transformer Action

- Transformer action \( \frac{\text{EMF}_S}{\text{EMF}_P} = \frac{N_S}{N_P} \)

Secondary

Primary

\( \Phi_B \sim B \)

\( B \sim I_1 \)

Def.: \( M_{12} = N_2 \frac{\Phi_B}{I_1} \)

What you need to know

- Transformers
  - Basic principle
  - Transformer in HVPS
  - Relationship between I,V,P on primary/secondary side
- Demos
  - Jacobs Ladder
  - Melting nail
Mutual Inductance

- Coupling is symmetric: $M_{12} = M_{21} = M$
- $M$ depends only on Geometry and Material
- Mutual inductance gives strength of coupling between two coils (conductors):
  \[ \text{EMF}_2 = -N_2 \frac{d\phi_B}{dt} = -M \frac{dI_1}{dt} \]
- $M$ relates $\text{EMF}_2$ and $I_1$ (or $\text{EMF}_1$ and $I_2$)
- Units: $[M] = \text{V}/(\text{A/s}) = \text{V s}/\text{A} = \text{H} ('\text{Henry}')$

Example: Two Solenoids

- Q: How big is $M = N_2 \frac{\phi_B}{I_1}$?
- A: $M = \mu_0 N_1 N_2 A/l$

Demo: Two Coils

- Signal transmitted by varying B-Field
- Coupling depends on Geometry (angle, distance)

Self Inductance

- L is also measured in [H]
- L connects induced EMF and variation in current:
  \[ \text{EMF} = -L \frac{dI}{dt} \]
- Remember Lenz’ Rule:
  Induced EMF will ‘act against’ change in current -> effective ‘inertia’
- Delay between current and voltage
What you need to know

- Inductance
  - Definition
  - Calculation for simple geometry
- Mutual Inductance
  - Definition
  - Calculation for simple geometry
  - Direction of induced EMF (depends only on dI/dt)
- Self Inductance
  - Definition
  - Calculation for simple geometry

RL Circuits

Kirchhoff's Rule: \( V_0 + \frac{\partial}{\partial t} = R \int \) \( \rightarrow V_0 = L \frac{dI}{dt} + R I \)

Q: What is \( I(t) \)?

**RL Circuits**

\[ I(t) = \frac{V_0}{R} \left[ 1 - \exp\left( \frac{-t}{\tau} \right) \right] \]
\[ V(t) = V_0 \exp\left( \frac{-t}{\tau} \right) \]

\( \tau = \frac{L}{R} \)

‘Back EMF’

- What happens if we move switch to position 2?
RL circuit

- L counteracts change in current both ways
  - Resists increase in I when connecting voltage source
  - Resists decrease in I when disconnecting voltage source
  - 'Back EMF'

- That’s what causes spark when switching off e.g. appliance, light

Energy Storage in Inductor

- Energy in Inductor
  - Start with Power \( P = V^2I = L \frac{dI}{dt} = \frac{dU}{dt} \)
  - \( \frac{dU}{L} = \frac{dI}{dt} \)
  - \( U = \frac{1}{2} LI^2 \)

- Where is the Energy stored?
  - Example: Solenoid (but true in general)
  - \( U/\text{Volume} = \frac{1}{2} B^2/\mu_0 \)

What you need to know

- Inductors
- \( I(t) \) in DC RL circuits
- Energy storage in inductors
- Practical use

RLC circuits

- Combine everything we know...
- Resonance Phenomena in RLC circuits
  - Resonance Phenomena known from mechanics (and engineering)
  - Great practical importance

Summary of Circuit Components

\[
\begin{align*}
\text{V} & : V(t) = V_0 \cos(\omega t) \\
\text{R} & : V_R = -IR \\
\text{L} & : V_L = -L \frac{dI}{dt} \\
\text{C} & : V_C = -Q/C = -1/C \int Idt
\end{align*}
\]

R,L,C in AC circuit

- AC circuit
  - \( I(t) = I_0 \sin(\omega t) \)
  - \( V(t) = V_0 \sin(\omega t + \phi) \)
  - \( \text{same } \omega! \)

- Relationship between V and I can be characterized by two quantities
  - Impedance \( Z = V_0/I_0 \)
  - Phase-shift \( \phi \)
AC circuit

\[ I(t) = I_0 \sin(\omega t) \]
\[ V(t) = V_0 \sin(\omega t + \phi) \]

Impedance \( Z = \frac{V_0}{I_0} \)
Phase-shift \( \phi \)

First: Look at the components

<table>
<thead>
<tr>
<th>Component</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>( V = IR )</td>
</tr>
<tr>
<td>Capacitor</td>
<td>( V = \frac{Q}{C} = \frac{1}{C} \int I dt )</td>
</tr>
<tr>
<td>Inductor</td>
<td>( V = L \frac{dI}{dt} )</td>
</tr>
</tbody>
</table>

\( Z = R \) \( \phi = 0 \)
\( Z = \frac{1}{(\omega C)} \) \( \phi = -\frac{\pi}{2} \)
\( Z = \omega L \) \( \phi = \frac{\pi}{2} \)

\( V \) and \( I \) in phase
\( V \) lags \( I \) by 90°
\( I \) lags \( V \) by 90°

RLC circuit

\[ V(t) \sim L \frac{dQ}{dt} - IR - \frac{Q}{C} = 0 \]

\[ L \frac{d^2Q}{dt^2} = -\frac{1}{C} Q - R \frac{dQ}{dt} + V \]

2nd order differential equation

Water
Spring Mass \( m \)

\( F_{ext} m \frac{d^2x}{dt^2} = -k x - f \frac{dx}{dt} + F_{ext} \)

Resonance

\( I_0 \)

\( I_{max} = \frac{V_0}{R} \)

\( \phi \)

\( \omega \)

\( \omega = \frac{1}{\sqrt{LC}} \)

Low Frequency
High Frequency
**RLC circuit**

\[ V_0 \sin(\omega t) = I_0 \left[ \frac{1}{\omega L} - \frac{1}{\omega C} \right] \cos(\omega t - \phi) + R \sin(\omega t - \phi) \]

Solution (requires two tricks):

\[ I_0 = \frac{V_0}{\left( \frac{1}{\omega L} - \frac{1}{\omega C} \right)^2 + R^2}^{1/2} = \frac{V_0}{Z} \]

\[ \tan(\phi) = \frac{\left( \frac{1}{\omega L} - \frac{1}{\omega C} \right)}{R} \]

\[ \Rightarrow \text{For } \omega L = \frac{1}{\omega C}, \text{ } Z \text{ is minimal and } \phi = 0 \]

\[ \text{i.e. } \omega_0 = \left( \frac{1}{LC} \right)^{1/2} \text{ Resonance Frequency} \]

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**Resonance**

- Practical importance
  - ‘Tuning’ a radio or TV means adjusting the resonance frequency of a circuit to match the frequency of the carrier signal

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**LC-Circuit**

- What happens if we open switch?

\[ -L \frac{dI}{dt} - \frac{Q}{C} = 0 \]

\[ L \frac{d^2Q}{dt^2} + \frac{Q}{C} = 0 \]

\[ \frac{d^2x}{dt^2} + \omega_0^2 x = 0 \]

\[ \text{Harmonic Oscillator!} \]

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**Energy in E-Field**

\[ \frac{1}{2} Q^2/C \]

\[ \frac{1}{2} L I^2 \]

**Potential Energy**

**Kinetic Energy**

**Oscillation**

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**LC-Circuit**

\[ d^2Q/dt^2 + 1/(LC) Q = 0 \]

\[ \omega_0^2 = 1/(LC) \]

\[ d^2x/dt^2 + k/m x = 0 \]

\[ \omega_0^2 = k/m \]

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**LC-Circuit**

- Total energy \( U(t) \) is conserved:
  - \( Q(t) \sim \cos(\omega t) \)
  - \( dQ/dt \sim \sin(\omega t) \)
  - \( U_L \sim (dQ/dt)^2 \sim \sin^2 \)
  - \( U_C \sim Q(t)^2 \sim \cos^2 \)
  - \( \cos^2(\omega t) + \sin^2(\omega t) = 1 \)
Electromagnetic Oscillations

- In an LC circuit, we see oscillations:
  - Energy in E-Field
  - Energy in B-Field

- Q: Can we get oscillations without circuit?
- A: Yes!
  - Electromagnetic Waves

What you need to know

- RLC Circuits
- How to obtain diff. equ (but not solve it)
- Definition of impedance, phase shift
- Phaseshift for C,R,L AC circuits
- Impedance, phase shift at resonance
- Limiting behavior of RLC circuit with frequency
- LC, RLC analogy with mechanical systems
- LC oscillations: Frequency, role of E,B energy

Displacement Current

- Ampere’s Law broken – How can we fix it?

\[ \mathbf{Q} = C \mathbf{V} \]

Displacement Current \( \mathbf{I_D} = \varepsilon_0 \frac{d\Phi_E}{dt} \)

Maxwell’s Equations

- Symmetry between E and B
  - although there are no magnetic monopoles
- Basis for radio, TV, electric motors, generators, electric power transmission, electric circuits etc
Maxwell’s Equations

\[
\oint_{\text{closed}} \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{inlet}}}{\varepsilon_0} \\
\oint_{\text{closed}} \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial \Phi_B}{\partial t} \\
\oint_{\text{closed}} \mathbf{B} \cdot d\mathbf{A} = 0 \\
\oint_{\text{closed}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{\text{inlet}} + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}
\]

- M.E.’s predict electromagnetic waves, moving with speed of light
- Major triumph of science

What you need to know

- Displacement current
  - Definition
  - Calculation for simple geometry
  - It’s not a current
- Maxwell’s equations
  - Meaning in words

Reminder on waves

- Types of waves
  - Transverse
  - Longitudinal
    - compression/decompression

Reminder on waves

For a travelling wave (sound, water)
Q: What is actually moving?

-> Energy!
- Speed of propagation: \( v = \lambda \cdot f \)
- Wave equation:

\[
\frac{\partial^2 D(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D(x,t)}{\partial t^2}
\]

Couples variation in time and space

Wave Equation

- Wave equation:

\[
\frac{\partial^2 D(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D(x,t)}{\partial t^2}
\]

Couples variation in time and space
- Speed of propagation: \( v = \lambda \cdot f \)
- We can derive a wave equation from Maxwell’s equations (NOT IN QUIZ)
Plane waves

- Example solution: Plane waves

\[ E_y = E_0 \cos(kz - \omega t) \]
\[ B_z = B_0 \cos(kz - \omega t) \]

with \( k = \frac{2\pi}{\lambda}, \omega = 2\pi f \) and \( f\lambda = c \).

E.M. Wave Summary

- \( \mathbf{E} \perp \mathbf{B} \) and perpendicular to direction of propagation
- Transverse waves
- Speed of propagation \( v = c = \frac{\lambda}{f} \)
- \( |E|/|B| = c \)
- E.M. waves travel without medium

What you need to know

- Waves
  - What is a wave?
  - Types of waves
  - Relationships between wavelength, frequency, wave speed
- E.M. waves
  - Properties
  - Connection to demos (speed, polarisation)
  - Relative direction of \( E, B, v \)

AMP Experiment

- Understand general idea/purpose
- Understand voltage dividers
- Understand need for negative feedback loop