Quiz 2

Name: (Last, First) __________________________ (please print).

Recitation number (circle one): 1 2 3

• Record all answers and show all work in this exam booklet. If you need extra space, use the back of the page.

• All scratch paper must be handed in with the exam, but will not be graded.

• This exam is closed book. You may use your handwritten notes if they are clearly labeled with your name and you hand them in with your exam.

• Whenever possible, try to solve problems using general analytic expressions. Plug in numbers only as a last step.

• Please make sure to answer all sub-questions.

• Good Luck!

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Question 1: Quick answers

(a). (5 pts) Consider the following reactions:
(A) ________ A 100 MeV photon decays into an electron-positron pair.
(B) ________ A neutron decays into an electron-positron pair and a photon
(C) ________ A neutron decays into a proton, an electron and a neutrino.
(D) ________ A proton decays into a neutron, a positron and a neutrino.
(E) ________ A neutron decays into a proton and a photon.

For each one, write one of the letters from the option list below.
L violates lepton number conservation
B violates baryon number conservation
P violates parity conservation
E violates energy-momentum conservation
Q violates charge conservation
N violates none of the above conservation laws

(b). (9 pts) Give each of the following quantities to the nearest power of 10 (don’t show calculations, being off by one power of 10 is OK):
(A) ________ Age of our universe when most He nuclei were formed
(B) ________ Age of our universe when hydrogen atoms formed
(C) ________ Age of our universe today
(D) ________ Number of stars in our Galaxy
(E) ________ Light travel time to closest star (Sun!) in minutes
(F) ________ Hydrogen binding energy in eV/c²
(G) ________ Electron mass in eV/c²
(H) ________ Neutron mass in eV/c²
(I) ________ Light travel time to 2nd closest star in years

(c). (9 pts) Indicate whether each of the following statements are true or false.
(A) TRUE / FALSE If our Universe is only X billion years old, then we can only see objects that are now less than X billion light years away
(B) TRUE / FALSE Space must be infinite, because it cannot end with a boundary without more space on the other side.
(C) TRUE / FALSE Leptons do not feel the strong interaction.
(D) TRUE / FALSE No experiment inside an isolated sealed lab in space can distinguish between whether it is uniformly accelerating or in a uniform gravitational field.
(E) TRUE / FALSE A clock by the ceiling runs faster than one by the floor.
(F) TRUE / FALSE Hubble’s law implies that the Big Bang was an explosion localized near the comoving position of our Galaxy.
(G) TRUE / FALSE The expansion of our galaxy is governed by the Friedmann equation.
(H) TRUE / FALSE Two galaxies can recede from each other faster than the speed of light.
(I) TRUE / FALSE We know that our entire observable universe was once at infinite density

(d). (2 pts) A tritium (H³) nucleus contains _____ up quarks and _____ down quarks.
In the Sun, one of the processes in the He fusion chain is \( p + p + e^- \rightarrow d + \nu \), where \( d \) is a deuteron. Make the approximations that the deuteron rest mass is \( 2m_p \), and that \( m_e \approx 0 \) and \( m_\nu \approx 0 \), since both the electron and the neutrino have negligible rest mass compared with the proton rest mass \( m_p \).

(a). For the arrangement shown in the figure, where (in the lab frame) the two protons have the same energy \( \gamma m_p \) and impact angle \( \theta \), and the electron is at rest, calculate the energy \( E_\nu \) of the neutrino in the rest frame of the deuteron in terms of \( \theta \), \( m_p \), and \( \gamma \).

(b). For the special case where the deuteron remains at rest in the lab frame and \( \theta = 30^\circ \), solve for \( \gamma \) and calculate the energy of all particles (the deuteron, the neutrino, one of the protons) in terms of the proton rest mass \( m_p \).
Question 3: Coulomb’s Law generalized

[25 Points]

In an inertial frame $S$, the position $\mathbf{r}_q$ of a point charge $q$ moves according to $\mathbf{r}_q(t) = v_z t$, i.e. with velocity $v$ in the $\hat{z}$-direction, passing the origin at $t = 0$. In the moving frame $S'$ where the charge is at rest at the origin, Coulomb’s law states that the electric field is

$$E' = A \frac{\mathbf{r}'}{r'^3},$$

where $A = q/4\pi\varepsilon_0$. Show that in the frame $S$, the electric field at $t = 0$ is

$$E = A \frac{(1 - \beta^2)}{\left(1 - \beta^2 \sin^2 \theta\right)^{3/2}} \frac{\mathbf{r}}{r^3},$$

where $\theta$ is the usual polar angle ($z = r \cos \theta, x^2 + y^2 = r^2 \sin^2 \theta$).
(a). **(10 pts)** Consider a particle coasting in the $r$-direction (i.e., with constant $\theta$ and $\phi$) in a flat FRW metric, with no non-gravitational forces acting on it. Use variational calculus to prove that $p \propto 1/a$ (here $p = m_0 \gamma u$ is its momentum and $u \equiv a \dot{r}$ is its velocity relative to nearby comoving observers).

(b). **(2 pts)** Given a), the value of $u$ in the limit $a \to 0$ is ________.  

(c). **(2 pts)** Given a), the value of $u$ in the limit $a \to \infty$ is ________.  

(d). **(2 pts)** Thus relative to comoving observers, your results show that an object without external forces in an expanding universe  
**(circle one)** REMAINS IN UNIFORM MOTION / SLOWS DOWN / ACCELERATES.  

(e). **(3 pts)** Starting with the answer from a), derive how the wavelength $\lambda$ of a photon depends on $a$. Your answer should be of the form $\lambda \propto \text{(function of } a\text{)}$. 

(f). **(6 pts)** Solve the Friedmann equation  

$$H^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{a^2}$$

to obtain a solution of the form $a(t) \propto \text{(function of } t\text{)}$ for the case where space is flat and the density is dominated by photons, and compute the age of the universe at the time when $H^{-1} = 30$ seconds.