1. Landau level for mass-less Dirac fermions

We have seen that the electron in the Graphene has a linear dispersion relation \( \epsilon(k) = v|k| \). Use the semi-classical approach to calculate the energy \( \epsilon_n \) of the \( n^{th} \) Landau level in the uniform magnetic field \( B \). Assume \( v = 1\text{eV} \times 1\text{Å}/\text{ℏ} \), find \( \epsilon_1 - \epsilon_0 \) in eV for a magnetic field of 30 Tesla. Can we see quantum Hall effect in Graphene at room temperature?

2. Hall conductance of electrons in a lattice

Consider a 2D spin-less non-interacting electron gas. The electron density is \( n \). A uniform magnetic field \( B = Bz \) is applied.

(a) Assume that the electrons have a dispersion \( \epsilon(k) = \frac{\hbar^2k^2}{2m} \). Use a classical approach to show that \( \rho_{xy} = +\frac{B}{\pi e}n \) and \( \rho_{xx} = 0 \), or \( \sigma_{xy} = -\frac{enB}{\pi} \) and \( \sigma_{xx} = 0 \). (Be careful about the signs and we assume the electron mean-free path to be \( l = \infty \).) Let \( S \) be the area enclosed by the Fermi surface. Show that

\[
\sigma_{xy} = -\frac{eSc}{4\pi^2B}
\]

The above formula is more general and works for an arbitrary dispersion \( \epsilon(k) \), as long as the Fermi surface forms a closed loop that encloses the occupied \( k \)-points.

(b) We like to apply the above classical result \( \sigma_{xy} = -\frac{eSc}{4\pi^2B} \) for electrons in square lattice with a lattice constant \( a \). The dispersion \( \epsilon(k) \) is given by

\[
\epsilon(k) = -2t[\cos(k_xa) + \cos(k_ya)]
\]

Find \( \sigma_{xy} \) as a function of electron density \( n \). (Hints: (a) The maximum density corresponds to a filled band. (b) \( \sigma_{xy} \) is known to be zero for a filled band. (c) \( \sigma_{xy} = -\frac{eSc}{4\pi^2B} \) is valid only if the Fermi surface forms a closed loop that encloses the occupied \( k \)-points. (d) For a nearly filled band, we may view the system as a system of holes.)

(c) (Optional) Guess how the above classical result should be modified if we include the quantum effect and impurity effect. Sketch the modified \( \sigma_{xy} \) as function of electron density \( n \) in the weak \( B \) field limit.

3. Diamagnetism – a simple way:

Consider a 2D spin-less non-interacting electron gas. The electron mass is \( m \) and the density is \( n \). Let \( E_{tot}(B) \) be the total ground state energy of the electrons.
(a) Find the values of $E_{tot}(B)$ when the magnetic field $B$ is such that the filling fraction $\nu$ is an integer.

(b) Find the values of $E_n = E_{tot}(B_n)$ when the magnetic field $B_n$ is such that the $n^{th}$ Landau level is half filled. Show that $E_n = c_0 + c_1 B_n^2$ and find the values of $c_0$ and $c_1$.

(c) Assume at $T \neq 0$, $\langle E_{tot} \rangle = c_0 + c_1 B^2$, find the corresponding magnetic susceptibility $\chi$.

4. Prob. 5 on page 319 of Kittle.

5. Prob. 6 on page 320 of Kittle.